ANIKET ' S

OPERATION

MATHEMATICS

CBSE (XII) – 2023
LEVEL – 0.1+0.2

(14th Revised Edition)

DATE : 1st JUNE, 2022

PREFACE

*As the examination pattern of **CBSE-XII** has been changed in such a way around **40%** questions are **1-Marker** and **2-Marker** questions.*

As per my observations, for most of the students, revision (preparation for exam) means direct reattempts to the questions of previously done books, very a few students revise the concepts and formulae unit wise.

*So, most of the students are ready to solve the **3–Marker** and **5–Marker** questions.*

*The above described process of revision was very fruitful for previous years, as **4–Marker (11 questions)** and **6–Marker (6 questions)** comprising **80 Marks** out of **100 Marks (80%).***

*So, this '**Operation Mathematics CBSE XII – 2023 (Level – 0.1+0.2)'** has been designed for providing target oriented preparation for **NCERT (0.1)** and **HOTS (0.2)** for **1–Marker** and **2–Marker** questions.*

*This package will help the students in scoring the wonder tons **90–100% Marks** in coming **CBSE XII–Exam.***

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1. Relation & Functions

***NCERT* (0.1)**

Q. 1. Give an example of a relation over the set $A = \{1, 2, 3\}$, which is

(i) symmetric and transitive but not reflexive. $Ans: \{(1,1), (2,2), (1,2), (2,1)\}$

(ii) transitive but neither reflexive nor symmetric. $Ans: \{(1,2), (1,3)\}$

(iii) reflexive and transitive but not symmetric. $Ans: \{(1,1), (2,2), (3,3), (1,2)\}$

Q. 2. Write down all possible equivalence relation over $A = \{1, 2, 3\}$ containing $(1, 2)$.

$Ans: \{(1,1), (2,2), (3,3), (1,2), (2,1)\}\ ;\ A \times A$

Q. 3. Write down all possible relations over $\{1, 2, 3\}$ containing $(1, 2)$ & $(2, 3)$ reflexive and transitive but not symmetric. $Ans: \{(1,1), (2,2), (3,3), (1,2), (2,3), (1,3)\};$

$\{(1,1), (2,2), (3,3), (1,2), (2,3), (1,3), (2,1)\};$

$\{(1,1), (2,2), (3,3), (1,2), (2,3), (1,3), (3,2)\}$

Q. 4. Write down all possible relations over $\{1, 2, 3\}$ containing $(1, 2)$ & $(1, 3)$ reflexive and symmetric but not transitive $Ans: \{(1,1), (2,2), (3,3), (1,2), (1,3), (2,1), (3,1)\}$

Q. 5. Let $A = \{-1,0,1,2\}, B = \{-4, -2,0,2\}$ and $f, g : A \to B$ be functions defined by $f(x) = x^2 - x, x \in A$ and $g(x) = 2\left|x - \frac{1}{2}\right| - 1, x \in A$. Show that f and g are equal.

Q. 6. Show that the Signum Function $f: R \to R$ given by $f(x) = \begin{cases} \frac{|x|}{x}\ ;\ x \neq 0 \\ 0\ ;\ x = 0 \end{cases}$ is neither one-one nor onto.

Q. 7. Show that an onto function $f: \{1, 2, 3\} \to \{1, 2, 3\}$ is always one – one.

Q. 8. Show that a one – one function $f: \{1, 2, 3\} \to \{1, 2, 3\}$ is always onto.

Q. 9. Show that the Greatest Integer Function $f: R \to R$ given by $f(x) = [x]$, is neither one-one nor onto. where $[x]$ is greatest integer $\leq x$,

Q. 10. Check the injectivity and surjectivity of $f: R \to R$ given by $f(x) = x^2$

$Ans: Neither\ injective\ nor\ surjective$

Q. 11. Check the injectivity and surjectivity of $f: N \to N$ given by $f(x) = x^2$

$Ans: Injective\ not\ surjective$

Q. 12. Check the injectivity and surjectivity of $f: Z \to Z$ given by $f(x) = x^2$

$Ans: Neither\ injective\ nor\ surjective$

Q. 13. Check the injectivity and surjectivity of $f: R \to R_+$ given by $f(x) = x^4$

$Ans: Not\ injective\ but\ surjective$

Q. 14. Find the number of all onto functions from the set $\{1, 2, 3, ..., n\}$ to itself. $Ans: n!$

Q. 15. Consider a function $f: \left[0, \frac{\pi}{2}\right] \to R$ given by $f(x) = sinx$ and $g: \left[0, \frac{\pi}{2}\right] \to R$ given by $g(x) = cosx$

Show that f and g are one – one but $f + g$ is not one – one.

***HOTS* (0.2)**

Q. 1. Find the smallest and largest possible equivalence relation over $A = \{1, 2, 3\}$

$Ans: smallest\ \{(1,1), (2,2), (3,3)\};\ largest = A \times A$

Q. 2. Find number all possible equivalence relation over $A = \{1, 2, 3\}$ $Ans: 5$

Q. 3. For the set $A = \{1, 2, 3\}$, define a relation R in the set A as follows: $R = \{(1,1), (2,2), (3,3), (1,3)\}$. Write the ordered pairs to be added to R to make it the smallest equivalence relation. $Ans: (3, 1)$

Q. 4. If $A = \{1, 2, 3\}$, then find the number of reflexive relation defined over set A $Ans: 64$

Q. 5. If $A = \{1, 2, 3\}$, then find the number of symmetric relation defined over set A $Ans: 64$

Q. 6. If $A = \{1, 2, 3\}$, then find the number of all possible relation defined over set A $Ans: 512$

Q. 7. For real numbers x and y, define xRy if and only if $x - y + \sqrt{2}$ is an irrational number. Then test the equivalence nature of relation R. $Ans: only\ reflexive$

Q. 8. Find the number of injective functions defined from $\{1, 2, 3\}$ to $\{a, b, c, d, e\}$ $Ans: 60$

Q. 9. Find the number of functions defined from $\{1, 2, 3\}$ to $\{a, b, c, d, e\}$ $Ans: 125$

Q. 10. If $f: A \to B$ is a bijective function such that $n(A) = 5$, then $n(B) = ?$ & the number of bijective functions will be? $Ans: n(B) = 5, 120$

Q. 11. If the function $f: R \to R$ given by $(x) = x + \sqrt{x^2}$. Check the bijective nature of function.
Ans: Neither injective nor surjective

Q. 12. If the function $f: R \to S$ given by $(x) = \frac{x}{x^2+1}$, is onto, find S. $Ans: \left[-\frac{1}{2}, \frac{1}{2}\right]$

Q. 13. If the function $f: [-1\ \ 1] \to S$ given by $(x) = \frac{x}{x+2}$, is invertible, find S. $Ans: \left[-1, \frac{1}{3}\right]$

Q. 14. If the function $f: R \longrightarrow (-1\ \ 1)$ is defined by $f(x) = \frac{x}{1+|x|}$ then find $f^{-1}\left(\frac{1}{3}\right)$ $Ans: \frac{1}{2}$

Q. 15. Let $A = \{1, 2, 3, -\ -\ -, n\}$ & $B = \{a, b\}$. Then find the number of surjective functions that can be defined from the set A to the set B. $Ans: 2^n - 2$

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2. Inverse Trigonometrical Functions

NCERT (0.1)

Q. 1. Evaluate: $tan\left\{2sin^{-1}\left(\frac{3}{5}\right)\right\}$ $Ans: x = \frac{24}{7}$

Q. 2. Simplify: $cos^{-1}\left\{cos\left(\frac{7\pi}{6}\right)\right\}$, $Ans: x = \frac{5\pi}{6}$

Q. 3. Find x if, $sin\left\{sin^{-1}x + cos^{-1}\left(\frac{1}{5}\right)\right\} = 1$, $Ans: x = \frac{1}{5}$

Q. 4. Simplify: $tan^{-1}\left(\frac{x}{\sqrt{a^2 - x^2}}\right)$, $Ans: sin^{-1}\left(\frac{x}{a}\right)$

Q. 5. Simplify: $tan^{-1}\left\{2cos\left(2sin^{-1}\left(\frac{1}{2}\right)\right)\right\}$, $Ans: x = \frac{\pi}{4}$

Q. 6. Simplify: $tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$ $Ans: tan^{-1}\left(\frac{a}{b}\right) - x$

Q. 7. Simplify: $tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$ $Ans: \frac{\pi}{4} - x$

Q. 8. Simplify: $cos^{-1}\{4x^3 - 3x\}$ $Ans: 3cos^{-1}x$

Q. 9. Evaluate : $sin\{tan^{-1}x\}$; $|x| \leq 1$ $Ans: \left[-\frac{1}{\sqrt{2}}\ \ \frac{1}{\sqrt{2}}\right]$

Q. 10. Evaluate : $tan^{-1}\sqrt{3} - cot^{-1}(-\sqrt{3})$ $Ans: -\frac{\pi}{2}$

Q. 11. Evaluate : $tan^{-1}\sqrt{3} - sec^{-1}(-2)$ $Ans: -\frac{\pi}{3}$

Q. 12. Evaluate : $tan^{-1}(1) + sin^{-1}\left(-\frac{1}{2}\right) + cos^{-1}\left(-\frac{1}{2}\right)$ $Ans: \frac{3\pi}{4}$

Q. 13. Show that: $tan^{-1}(x) + tan^{-1}\left(\frac{2x}{1-x^2}\right) = 3tan^{-1}(x)$; $|x| \leq \frac{1}{\sqrt{3}}$

Q. 14. Prove that : $\frac{9\pi}{8} - \frac{9}{4}sin^{-1}\left(\frac{1}{3}\right) = \frac{9}{4}sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$

Q. 15. Show that: $tan^{-1}(\sqrt{x}) = \frac{1}{2}cos^{-1}\left(\frac{1-x}{1+x}\right)$

HOTS (0.2)

Q. 1. Simplify: $tan\left(\frac{1}{2}cos^{-1}\left(\frac{3}{11}\right)\right)$ $Ans: \frac{2}{\sqrt{7}}$

Q. 2. If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = 3\pi$, then value of $x(y+z) + y(z+x) + z(x+y)$ $Ans: 6$

Q. 3. If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = 0$, then value of $x + y + z$ $Ans: 3$

Q. 4. If $tan^{-1}x + tan^{-1}y + tan^{-1}z = \pi$, then prove that $x + y + z = xyz$

Q. 5. Evaluate: $tan^2(sec^{-1}(2)) + \cot^2(cosec^{-1}(3))$ $Ans: 11$

Q. 6. Find x, if $4\,sin^{-1}x + cos^{-1}x = \pi$, $Ans: \frac{1}{2}$

Q. 7. If $tan^{-1}x + tan^{-1}y + tan^{-1}z = \frac{\pi}{2}$, then prove that $xy + yz + zx = 1$

Q. 8. If, $y = cot^{-1}(\sqrt{\cos x}) - tan^{-1}(\sqrt{\cos x})$, prove that $\sin y = tan^2\left(\frac{x}{2}\right)$.

Q. 9. Evaluate: $4(cot^{-1}3 + cosec^{-1}\sqrt{5})$ $Ans: \pi$

Q. 10. If in any triangle $ABC, A = tan^{-1}2, B = tan^{-1}3$. Find $\angle C$ — $Ans: \frac{\pi}{4}$

Q. 11. Evaluate: $cos^{-1}(cos\, 4)$ — $Ans: 2\pi - 4$

Q. 12. Evaluate: $sin^{-1}(sin\, 4)$ — $Ans: \pi - 4$

Q. 13. Find x, if $sin^{-1}x \le cos^{-1}x$ — $Ans: \left[-1, \frac{1}{\sqrt{2}}\right]$

Q. 14. Find x, if, $tan(cos^{-1}x) = sin(tan^{-1}2)$ — $Ans : \pm\frac{\sqrt{5}}{3}$

Q. 15. Simplify: $sin^{-1}\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)$ — $Ans: \frac{\pi}{12}$

Q. 16. Simplify: $sin^{-1}\left\{\frac{2sin\, x + 3cos\, x}{\sqrt{13}}\right\}$ — $Ans : x + tan^{-1}\left(\frac{3}{2}\right)$

Q. 17. Evaluate: $cos^{-1}x - cos^{-1}\left\{\frac{x}{2} + \frac{1}{2}\sqrt{3-3x^2}\right\}; 0 \le x \le \frac{1}{2}$ — $Ans: \frac{\pi}{3}$

Q. 18. Find domain of function $cos^{-1}(x^2 - 4)$ — $Ans: [-\sqrt{5}\ \ -\sqrt{3}] \cup [\sqrt{3}\ \ \sqrt{5}]$

Q. 19. Find principle range of $cosec^{-1}x,\ |x| \le 2$ — $Ans: \left[-\frac{\pi}{2}\ \ -\frac{\pi}{6}\right] \cup \left[\frac{\pi}{6}\ \ \frac{\pi}{2}\right]$

Q. 20. Evaluate: $sin\left\{\frac{1}{4}sin^{-1}\left(\frac{\sqrt{63}}{8}\right)\right\}$ — $Ans: \frac{1}{2\sqrt{2}}$

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3. Matrices

NCERT (0.1)

Q. 1. Construct a 3×3 matrix with $a_{ij} = \frac{|j - 3i|}{2}$ — $Ans: \begin{bmatrix} 1 & \frac{1}{2} & 0 \\ \frac{5}{2} & 2 & \frac{3}{2} \\ 4 & \frac{7}{2} & 3 \end{bmatrix}$

Q. 2. Find the value of a, b, c and d, if $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$ — $Ans: 1, 2, 3, 4$

Q. 3. Find the value of x and y if $\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$ — $Ans: No\ values.$

Q. 4. Find the value of 'x' if, $[x \quad -5 \quad -1]\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$ — $Ans: -1$

Q. 5. If A is any square matrix so that, $A^2 = A$, then find $(I + A)^3 - 7A$. — $Ans: I$

Q. 6. If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$, then value of $\alpha^2 + \beta\gamma$ is — $Ans: 1$

Q. 7. Find the matrix which is symmetric and skew-symmetric together. — $Ans: O(square)$

Q. 8. Find the value of x, y, z if the matrix $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfies the condition $A^TA = I_3$

$Ans: x = \pm\frac{1}{\sqrt{2}}, y = \pm\frac{1}{\sqrt{6}}, z = \pm\frac{1}{\sqrt{3}}$

Q. 9. Show that for any invertible matrix A, $(A^{-1})^{-1} = A$

Q. 10. If A and B are symmetric matrices of same order then check whether that $AB - BA$ is symmetric or skew – symmetric matrix. — $Ans: Skew - symmetric$

Q. 11. Show that, if inverse of a square matrix exists, then it is unique.

Q. 12. If A & B are invertible matrices of the same order, then prove that $(AB)^{-1} = B^{-1}A^{-1}$

Q. 13. If A is any square matrix. Show that $A + A^T$ is symmetric & $A - A^T$ is skew-symmetric matrices.

Q. 14. If $A = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}$ and $B = [1 \quad 3 \quad -6]$, then verify $(AB)^T = B^T.A^T$

Q. 15. If A, B & C are matrix of order $p \times k$, $3 \times k$ & $n \times 3$ respectively, the restriction on n, k & p so that $AB + CB$ will be defined is — $Ans: p = n, k = 3$

***HOTS* (0.2)**

Q. 1. Show that the diagonal elements of a skew symmetric matrix are always zero.

Q. 2. If A is any invertible matrix prove that $(A^{-1})^T = (A^T)^{-1}$

Q. 3. If $A = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$ then find $I + 2A + 3A^2 + ---\infty$ $\quad Ans: \begin{bmatrix} 5 & 2 \\ -8 & -3 \end{bmatrix}$

Q. 4. If A & B are two matrices such that $AB = A$ & $BA = B$ then B^2 is equals to $\quad Ans: B$

Q. 5. Find $a + b + c$, if $\begin{bmatrix} a & x & y \\ x & b & c \\ y & c & 5 \end{bmatrix}$ is a scalar matrix $\quad Ans: 10$

Q. 6. If $A = [3 \quad 5]$ & $B = [7 \quad 3]$ then find the matrix C such that $AC = BC$. $Ans: \begin{bmatrix} x \\ 2x \end{bmatrix}, \begin{bmatrix} x & y \\ 2x & 2y \end{bmatrix} ---$

Q. 7. If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ satisfies $(A + B)^2 = A^2 + B^2$. Find the value of a and b.

Q. 8. For the matrix $A = \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$. Show that $A^{-1} = A$

Q. 9. Give an example of matrices A, B & C such that $AB = AC$; $A \neq O, B \neq C$

Q. 10. Let the matrix A is skew symmetric. Prove that A^n is a symmetric matrix, if n is an even integer.

Q. 11. Let the matrix $A = [a_{ij}]_{m\times m}$ is skew symmetric. Find $\sum_i^m \sum_j^m a_{ij}$ $\quad Ans: 0$

Q. 12. If $A = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}, i = \sqrt{-1}$ find $(A)^{4n}, n \in Natural$ $\quad Ans = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Q. 13. Let the matrix $A = \frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$ prove that $A^{-1} = A^T$

Q. 14. If $\begin{bmatrix} \cos\left(\frac{2\pi}{7}\right) & -\sin\left(\frac{2\pi}{7}\right) \\ \sin\left(\frac{2\pi}{7}\right) & \cos\left(\frac{2\pi}{7}\right) \end{bmatrix}^n = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find least positive integral value of n $\quad Ans: 7$

Q. 15. If A is any non – singular square matrix, then prove that $(A^{-1})^{-1} = A$

Q. 16. If A is 3×3 invertible matrix, then show that $(kA)^{-1} = \frac{1}{k}(A^{-1})$

Q. 17. Under what condition on matrix $A, AB = AC \Rightarrow B = C$ $\quad Ans: |A| \neq 0$

Q. 18. If either $A = O$ or $B = O$, then $AB = O$, but the converse need not be true. Justify your answer.

Q. 19. If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$. Then find A^n $\quad Ans: \begin{bmatrix} a^n & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a^n \end{bmatrix}$

Q. 20. If $A^2 + I = O$, then find the matrix $[A]_{2\times 2}$. $\quad Ans: \begin{bmatrix} \pm i & 0 \\ 0 & \pm i \end{bmatrix}$

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4. Determinants

***NCERT* (0.1)**

Q. 1. If $D = \begin{vmatrix} 1 & \sin x & 1 \\ -\sin x & 1 & \sin x \\ -1 & -\sin x & 1 \end{vmatrix}$ prove that $2 \leq D \leq 4, \forall x \in real$

Q. 2. If A is any non – singular square matrix, then prove that $A^{-1} = \frac{1}{|A|} Adj(A)$

Q. 3. If the area of the $\Delta ABC = 35\ sq.units$. Using determinants find the values of k, if the vertices of the triangle ABC has coordinates $A(2,-6), B(5,4)$ & $C(k,4)$ $\quad Ans: -2, 12$

Q. 4. Find the values of x, if the points $A(2,-6), B(5,4)$ & $C(k,4)$ are collinear. $\quad Ans: 5$

Q. 5. If A is a square matrix of order 3×3 with, $|A| = 3$, then find the value of $|Adj(A)|$ $\quad Ans: 9$

Q. 6. If A is a 3×3 matrix, $|A| \neq 0$ and $|3A| = k|A|$ then find the value of k. $\quad Ans: 27$

Q. 7. Using determinants find the equation of the straight line joining $A(2,-6)$ & $B(5,4)$.

$Ans: 10x - 3y = 38$

Q. 8. Find the area of the triangle whose vertices are $(3,8),(-4,2)$ & $(5,1)$. $Ans: \frac{61}{2}$

Q. 9. Find the values of x, if the matrix $A = \begin{bmatrix} -2 & 1 & x \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{bmatrix}$ is non – invertible. $Ans: 4$

Q. 10. Using Cofactors of elements of third column, prove that $\begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix} = (x-y)(y-z)(z-x)$

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***HOTS* (0.2)**

Q. 1. Show that the determinant of a skew symmetric matrix of odd order is zero

Q. 2. If the value of a third order determinant is 12, then find the value of the determinant formed by replacing each element by its co-factor. $Ans: 144$

Q. 3. If A is any square matrix of order 3×3 & $|A| = 5$ find $|adj(adj\ (A))|$ $Ans: 625$

Q. 4. If A & B are square matrices of same order and $AB = BA = I$, then show that they are non – singular.

Q. 5. Find $|A^{-1}|$ if for the square matrix A of order 3×3, $|adj\ (A)| = 16$ $Ans: \pm\frac{1}{4}$

Q. 6. If A is a diagonal matrix of order 3×3 with $a_{11} = 2, a_{22} = 3$ & $a_{33} = 4$ then find $|A|$ $Ans: 24$

Q. 7. If A & B are square matrices of order 3×3 such that $|A| = -2, |B| = 3$ prove that $|3AB^{-1}| = -18$

Q. 8. If A is a square matrix of order 3×3 with, $|A| = 3$, then find the value of $|3A^{-1}|$ $Ans: 9$

Q. 9. Evaluate : $\begin{vmatrix} 0 & 23 & -32 \\ -23 & 0 & -29 \\ 32 & 29 & 0 \end{vmatrix}$ $Ans: 0$

Q. 10. Evaluate $\begin{vmatrix} 9log_3 2 & \frac{1}{2}log_2 3 \\ log_3 8 & log_2 3 \end{vmatrix}$ $Ans: \frac{15}{2}$

Q. 11. If $A = diag[a_1\ a_2\ a_3 - - - a_n]$, prove that $|A^n| = (a_1.a_2.a_3.---.a_n)^n$

Q. 12. If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ then evaluate, $a_{11}A_{21} + a_{12}A_{22} + a_{13}A_{23}$ $Ans: 0$

Q. 13. Evaluate $\begin{vmatrix} log_3 512 & log_4 3 \\ log_3 8 & log_4 9 \end{vmatrix}$ $Ans: \frac{15}{2}$

Q. 14. If the matrix $A = \begin{bmatrix} x-1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{bmatrix}$ is singular find the value of x. $Ans: -1, 2$

Q. 15. If a, b & c are three sides of the triangle ABC with $A(x_1,y_1), B(x_2,y_2)$ & $C(x_3,y_3)$, then show that

$$\begin{vmatrix} x_1 & y & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2 = \frac{(a+b+c)(a+b-c)(c+a-b)(b+c-a)}{4}$$

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5. Differentiations

***NCERT* (0.1)**

Q. 1. If the function $f(x) = \begin{cases} \frac{|x|}{x} & ; x < 0 \\ k & ; x \geq 0 \end{cases}$ is continuous at $x = 0$, then find the value of k. $Ans: -1$

Q. 2. If the function $f(x) = cot\ x$ is continuous, then find the value of x. $Ans: R - n\pi, n \in Z$

Q. 3. If the function $f(x) = \frac{x^2-25}{x-5}$ is continuous, then find the value of x $Ans: R - \{5\}$

Q. 4. Find the points of non differentiability of the function $f(x) = [x], x \in (0\ \ 3)$ $Ans: 1, 2$

Q. 5. Find the number of points of non differentiability of the function $f(x) = |x| + |x-1|$ $Ans: 2$

Q. 6. For what values of λ, the function $f(x) = \begin{cases} \lambda(x^2 - 2x); & x \le 0 \\ 4x + 1 & ; x > 0 \end{cases}$ is continuous at $x = 0$ Ans : No λ

Q. 7. Find the relationship between a and b if the function $f(x) = \begin{cases} ax + 1\,; x \le 3 \\ bx + 3\,; x > 3 \end{cases}$ is continuous at $x = 3$

$Ans: 3(a + b) = 2$

Q. 8. Discuss the continuity of the function defined by $f(x) = \begin{cases} 2 + x\,; x > 0 \\ 2 - x\,; x < 0 \end{cases}$

$Ans: continuous\ in\ R - \{0\}$

Q. 9. If $y = tan^{-1}x$, prove that $\frac{dy}{dx} = \frac{1}{1+x^2}$

Q. 10. If $y = cos^{-1}x$, then prove that $\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}$

Q. 11. If $= cos^{-1}\left\{\frac{2x}{1+x^2}\right\}; -1 < x < 1$ then prove that $\frac{dy}{dx} = -\frac{2}{1+x^2}$

Q. 12. If, $y = sec^{-1}\left(\frac{1}{1-2x^2}\right)$ then prove that $\frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}$

Q. 13. If $y = tan^{-1}\left\{\frac{3x - x^3}{1-3x^2}\right\}; -1 < x < 1$ then prove that $\frac{dy}{dx} = \frac{3}{1+x^2}$

Q. 14. If $y = 2\sqrt{cot(x^2)}$, prove that $\frac{dy}{dx} = \frac{-2\sqrt{2}\,x}{sin(x^2)\sqrt{sin(2x^2)}}$

Q. 15. If $= a\left(cos\,x + log\left\{tan\left(\frac{x}{2}\right)\right\}\right)$, prove that $\frac{dy}{dx} = a\,cos\,x\,.\,cot\,x$

Q. 16. If $y = log_7 log(sinx)$, then prove that $\frac{dy}{dx} = \frac{cot\,x}{log\,7}$

Q. 17. If $y = cos^{-1}x$, then find $\frac{d^2y}{dx^2}$ in terms of y alone. $Ans: -cosec^2\,y.cot\,y$

Q. 18. If $x = \sqrt{a^{sin^{-1}t}}$ and $y = \sqrt{a^{cos^{-1}t}}$ prove that $\frac{dy}{dx} = -\frac{y}{x}$.

Q. 19. If $y = sin^2y + cos\,xy = \pi$, prove that $\frac{dy}{dx} = \frac{y\,sin(xy)}{sin\,2y - x\,sin(xy)}$

Q. 20. If $y = cos(x^3).sin^2(x^5)$, prove that $\frac{dy}{dx} = 5x^4cos(x^3).sin(2x^5) - 3x^2sin(x^3).sin^2(x^5)$

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***HOTS* (0.2)**

Q. 1. If, $f(x) = \frac{1}{2+x}$ find the points of discontinuities of $f(f(x))$

Q. 2. If, $f(x) = \frac{1}{log\,|x|}$ find the points of discontinuities of $f(x)$

Q. 3. Find the domain of continuity of the function $f(x) = sin^{-1}x - [x]$ is

Q. 4. If $e^x + e^y = e^{x+y}$, prove that $\frac{dy}{dx} = -e^{y-x}$

Q. 5. If $f(x) = |cos\,x - sinx|$, find $\left.\frac{dy}{dx}\right)_{x=\frac{\pi}{3}}$

Q. 6. If $x^2 + y^2 = t - \frac{1}{t}$ and $x^4 + y^4 = t^2 + \frac{1}{t^2}$ prove that $\frac{dy}{dx} = \frac{1}{x^3y}$

Q. 7. If $y = sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right) + sec^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)$ find $\frac{dy}{dx}$

Q. 8. If, $y = cos^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right)$ find $\frac{dy}{dx}$

Q. 9. If $y = x^{x^x}$. Find $\frac{dy}{dx}$

Q. 10. If $y^x.x^y = x^x$. Find $\frac{dy}{dx}$

Q. 11. If $y = tan^{-1}\left(\frac{x-\sqrt{x}}{1+x^{3/2}}\right)$, find $\frac{dy}{dx}$

Q. 12. If $y = tan^{-1}x + tan^{-1}\left(\frac{4x}{1-5x^2}\right)$, find $\frac{dy}{dx}$

Q. 13. If $sin\,x = \frac{2t}{1+t^2}$ & $tan\,y = \frac{2t}{1-t^2}$, then find $\frac{dy}{dx}$

Q. 14. If $y^3 = 3ax^2 - x^3$ then prove that $\frac{d^2y}{dx^2} = \frac{-2\,a^2x^2}{y^5}$

Q. 15. If $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then show that $\frac{d^2y}{dx^2} = -\frac{b^4}{a^2y^3}$

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6. Applications of Derivatives

***NCERT* (0.1)**

Q. 1. The total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$. Find the marginal revenue, when $x = 5$. *Ans*: 66

Q. 2. An edge of a variable cube is increasing at the rate of $3\ cm/s$. How fast is the volume of the cube increasing when the edge is $10\ cm$ long? *Ans*: $900\ cm^3/s$

Q. 3. A stone is dropped into a quiet lake and waves move in circles at a speed of $4\ cm/s$. At the instant, when the radius of the circular wave is $10\ cm$, how fast is the enclosed area increasing? *Ans*: $80\pi\ cm^2/s$

Q. 4. A balloon, which always remains spherical has a variable radius. Find the rate at which its volume is increasing with the radius when the later is $10\ cm$. *Ans*: $400\pi\ cm^3/cm$

Q. 5. A balloon, which always remains spherical, has a variable diameter $\frac{3}{2}(2x+1)$. Find the rate of change of its volume with respect to x. *Ans*: $\frac{27\pi}{8}(2x+1)^2$

Q. 6. A balloon, which always remains spherical on inflation, is being inflated by pumping in $900\ cm^3$ of gas per second. Find the rate at which the radius of the balloon increases when the radius is $15\ cm$. *Ans*: $\frac{1}{\pi} cm/s$

Q. 7. Find maximum and the minimum value of $3x^4 - 8x^3 + 12x^2 - 48x + 25, x \in [0,\ 3]$ *Ans*: $25, -39$

Q. 8. Find the interval if the function $f(x) = log\,x$ is strictly increasing. *Ans*: $(0, \infty)$

Q. 9. Find the intervals if the function $f(x) = sin\,x + cos\,x$, is strictly increasing. *Ans*: $\left(0,\ \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right)$

Q. 10. Find maximum and minimum values of $2x^3 - 15x^2 + 36x + 1, x \in [1,\ 5]$ *Ans*: $56,\ 24$

Q. 11. Find the maximum and minimum value of the function $f(x) = x^3 + x^2 + x + 1$ *Ans*: *doesn't exist*

Q. 12. Find the values of x if, $y = [x(x-2)]^2$ is an increasing function. *Ans*: $(0,1) \cup (2, \infty)$

Q. 13. Find the least value of a such that the function f given by $f(x) = x^2 + ax + 1$ is strictly increasing on (1, 2). *Ans*: -2

Q. 14. Prove that the function $f(x) = x^2 - x + 1$, is neither increasing nor decreasing on $(-1\ 1)$.

Q. 15. Find the critical point of the function $f(x) = \frac{log\,x}{x}$ *Ans*: $x = e$

Q. 16. Find the intervals in which the function $f(x) = x + \frac{1}{x}$ is (i) increasing (ii) decreasing *Ans*: $(i)(-\infty, -1) \cup (1, \infty)$ $(ii)(-1, 1)$

Q. 17. Find the maximum and minimum value of $f(x) = sin\,x + cos\,x\,; x \in [0,\ \pi]$ *Ans*: $\sqrt{2}, -1$

Q. 18. If the function $x^4 - 62x^2 + ax + 9$ attains its maximum value at $x = 1 \in [0, 2]$. Find the value of a. *Ans*: 120

Q. 19. Find absolute maximum and minimum values of the function $f(x) = 12x^{4/3} - 6x^{1/3}, x \in [-1,\ 1]$. *Ans*: $18, -\frac{9}{4}$

Q. 20. Find the interval in which the function $f(x) = sin\,3x\,, x \in \left[0,\ \frac{\pi}{4}\right]$ is increasing. *Ans*: $\left(0,\ \frac{\pi}{6}\right)$

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***HOTS* (0.2)**

Q. 1. Find the rate of change of volume of a sphere with respect to its surface area, when the radius is $4\ cm$.
$Ans: 2\ cm^3/cm^2$.

Q. 2. Find the length of the edge of the cube such that the rate of increase of its surface is same as the rate of increase of its volume. $Ans: 4\ units$

Q. 3. If the area of a circle is increasing at a constant rate, show that the rate of increase in the perimeter varies inversely as radius.

Q. 4. A ladder is inclined to a wall making an angle $\frac{\pi}{6}$ with it. A man is ascending the ladder at the rate of $3m/sec$. How fast is he approaching the wall? $Ans: \frac{3}{2}m/s$

Q. 5. Find the interval in which the function $f(x) = x^x, x > 0$ is increasing. $Ans: \left[\frac{1}{e}\ \ \infty\right[$

Q. 6. If $y = alog\ x + bx^2 + x$ has extreme value at $x = -1, 2$ then prove that $a + b = \frac{1}{10}$

Q. 7. Show that $f(x) = x^3 - 6x^2 + 18x + 5$ is an increasing function for all $x \in Real$

Q. 8. Find the interval in which the function $f(x) = \sin^4 x + \cos^4 x, x \in \left[0\ \ \frac{\pi}{2}\right]$ is increasing $Ans: \left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

Q. 9. Find the interval if the function $f(x) = cot^{-1}(sin\,x + cos\,x), x \in \left[0, \frac{\pi}{2}\right]$ is increasing. $Ans: \left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

Q. 10. Find the maximum and minimum value of the function $f(x) = |4sinx + 3\cos x - 2|\ ;\ x \in real$
$Ans: Max\ value = 7, Min\ value = 0$

Q. 11. Find the maximum and minimum value of the function $f(x) = 2x^3 - 6x^2 + 6x + 5$
$Ans: doesn't\ exist$

Q. 12. Find the critical points of the function $f(x) = 12x^{4/3} - 6x^{1/3}$ $Ans: 0, \frac{1}{8}$

Q. 13. Find the maximum and minimum value of $|sin\,3x - 4|$ $Ans: Max\ value = 5, Min\ value = 3$

Q. 14. The critical point of the function $f(x) = 4x^3 - 18x^2 + 27x - 7$ is point of inflection. Justify

Q. 15. Find the least value of the function $f(x) = ax + \frac{b}{x}\ ; a, b, x > 0$ $Ans: 2\sqrt{ab}$

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7. Indefinite Integrations

***NCERT* (0.1)**

Q. 1. Prove that : $\int \frac{dx}{\sqrt{x} - x} = -2\,log\left|1 - \sqrt{x}\right| + C$

Q. 2. Prove that : $\int \frac{x - 1}{\sqrt{x^2 - 1}}\,dx = \sqrt{x^2 - 1} - \log\left|x + \sqrt{x^2 - 1}\right| + C$

Q. 3. Prove that : $\int \frac{dx}{sin\,x.\,cos^3 x} = log\left|tan\,x\right| + \frac{tan^2 x}{2} + C$

Q. 4. Prove that : $\int \frac{e^{5logx} - e^{4logx}}{e^{3logx} - e^{2logx}}\,dx = \frac{x^3}{3} + C$

Q. 5. Prove that : $\int tan^4 x dx = \frac{tan^3 x}{3} - tan\,x + x + C$

Q. 6. Prove that : $\int sin^4 x dx = \frac{3x}{8} - \frac{sin\,2x}{4} + \frac{sin\,4x}{32} + C$

Q. 7. Prove that : $\int tan^3 2x.\,sec\,2x\,dx = \frac{sec^3 2x}{6} - \frac{sec\,2x}{2} + C$

Q. 8. Prove that : $\int \frac{dx}{1 + tan\,x} = \frac{x}{2} + \frac{1}{2}log\left|cos\,x + sin\,x\right| + C$

Q. 9. Prove that : $\int \frac{x^2}{1 - x^6}\,dx = \frac{1}{6}\,log\left|\frac{1 + x^3}{1 - x^3}\right| + C$

Q. 10. Prove that : $\int e^{3\,log\,x}(1 + x^4)^{-1}dx = \frac{1}{4}\log|1 + x^4| + C$

Q. 11. Prove that : $\int \frac{\sqrt[4]{x^4-x}}{x^5}\,dx = \frac{4}{15}\left(1-\frac{1}{x^3}\right)^{5/4} + C$

Q. 12. Prove that : $\int \frac{\sin x}{\sin(x+a)}\,dx = x\cos a - \sin a \log|\sin(x+a)| + C$

Q. 13. Prove that : $\int \frac{e^{2x}-1}{e^{2x}+1}\,dx = \log|e^x + e^{-x}| + C$

Q. 14. Prove that : $\int \frac{\sqrt{\tan x}}{\sin x.\cos x}\,dx = 2\sqrt{\tan x} + C$

Q. 15. Prove that : $\int \sin^3 x.\cos^2 x\,dx = -\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} + C$

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***HOTS* (0.2)**

Q. 1. Prove that : $\int \sqrt{\frac{x}{x^3+a^3}}\,dx = \frac{2}{3}\log\left|x^{3/2} + \sqrt{x^3+a^3}\right| + C$

Q. 2. Prove that : $\int \{\sin(\log x) + \cos(\log x)\}dx = x\sin(\log x) + C$

Q. 3. Prove that : $\int \frac{\sin 3x}{\sin 2x\,.\,\sin 5x}\,dx = \frac{\log(\sin 2x)}{2} - \frac{\log(\sin 5x)}{5} + C$

Q. 4. Prove that : $\int \sqrt{\sec x - 1}\,dx = -\log\left|(2\cos x + 1) + 2\sqrt{\cos x + \cos^2 x}\right| + C$

Q. 5. Prove that : $\int \frac{\sin x}{\sin 3x}\,dx = \frac{1}{2\sqrt{3}}\log\left|\frac{\sin x + \sqrt{3}\cos x}{\sin x - \sqrt{3}\cos x}\right| + C$

Q. 6. Prove that : $\int e^{\tan^{-1}x}\left(\frac{1+x+x^2}{1+x^2}\right)dx = xe^{\tan^{-1}x} + C$

Q. 7. Prove that : $\int \frac{dx}{\sin^{3/4}x\,.\,\cos^{5/4}x} = 4\sqrt[4]{\tan x} + C$

Q. 8. Prove that : $\int \frac{\cos x - \cos 2x}{1-\cos x}\,dx = x + 2\sin x + C$

Q. 9. Prove that : $\int \frac{dx}{x.\log x.\log(\log x)} = \log(\log(\log x)) + C$

Q. 10. Prove that : $\int (\cot x.\cot 2x - \cot 2x.\cot 3x - \cot 3x.\cot x)dx = x + C$

Q. 11. Prove that : $\int \left(\frac{\tan x + \tan^3 x}{1+\tan x}\right)dx = \tan x + \log|1+\tan x| + C$

Q. 12. Prove that : $\int \frac{x^9}{(1+4x^2)^6}\,dx = \log x - \frac{1}{2}\log(4x^2+1) + C$

Q. 13. Prove that : $\int 5^{5^x}.5^x\,dx = \frac{5^{5^x}}{(\log 5)^2} + C$

Q. 14. Prove that : $\int \frac{dx}{1+\sin x} = \tan x - \sec x + C$

Q. 15. Prove that : $\int \left(\frac{a^x+b^x}{c^x}\right)dx = \frac{(a/c)^x}{\log a - \log c} + \frac{(b/c)^x}{\log b - \log c} + C$

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8. Definite Integrations

***NCERT* (0.1)**

Q. 1. Prove that : $\int_1^2 \frac{dx}{x\sqrt{x^2-1}} = \frac{\pi}{3}$

Q. 2. Prove that :: $\int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2}\right)e^{2x}dx = \frac{e^2(e^2-2)}{4}$

Q. 3. Prove that : $\int_0^{\pi/2} sin^3 x dx = \frac{2}{3}$

Q. 4. Prove that : $\int_0^{2\pi} \cos^5 x\, dx = 0$

Q. 5. Prove that : $\int_{\pi/3}^{\pi/2} cosec\, x\, dx = \frac{1}{2} log\, 3$

Q. 6. Prove that : $\int_0^{\pi/2} \sqrt{sin\, x}\,.\cos^5 x\, dx = \frac{64}{231}$

Q. 7. Prove that : $\int_0^1 sin^{-1}\left(\frac{2x}{1+x^2}\right) dx = \frac{\pi}{2} - log\, 2$

Q. 8. Prove that : $\int_{1/3}^1 \frac{\sqrt[3]{x-x^3}}{x^4} dx = 6$

Q. 9. Prove that : $\int_0^2 \frac{6x+3}{x^2+4} dx = 3\, log\, 2 + \frac{3\pi}{8}$

Q. 10. Prove that : $\int_0^1 sin^{-1} x\, dx = \frac{\pi}{2} - 1$

Q. 11. Prove that : $\int_0^{\pi/4} 2tan^3 x dx = 1 - log\, 2$

Q. 12. Prove that : $\int_0^1 tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right) dx = 0$

Q. 13. Prove that : $\int_1^3 \frac{dx}{x^2(x+1)} = \log\left(\frac{2}{3}\right) + \frac{2}{3}$

Q. 14. Prove that : $\int_{\pi/3}^{\pi} \left(\frac{1-\sin x}{1-\cos x}\right) e^x\, dx = e^{\pi/2}$

Q. 15. Prove that: $\int_{-\pi/2}^{\pi/2} (x^3 + x\cos x + \tan^5 x + 1)\, dx = \pi$

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<u>HOTS (0.2)</u>

Q. 1. Prove that : $\int_2^3 x(5-x)^n x dx = \frac{5}{n+1}(3^{n+1} - 2^{n+1}) - \frac{1}{n+2}(3^{n+2} - 2^{n+2})$

Q. 2. Prove that : $\int_0^{\pi/4} \sqrt{1 - sin\, 2x}\, dx = \sqrt{2} - 1$

Q. 3. Prove that : $\int_0^1 tan^{-1} x\, dx = \frac{\pi}{4} - \frac{1}{2} log\, 2$

Q. 4. Prove that : $\int_0^{\sqrt{2}} [x]\, dx = \sqrt{2} - 1$

Q. 5. Prove that : $\int_4^9 \frac{\sqrt{x}}{(30 - x\sqrt{x})^2} dx = \frac{19}{99}$

Q. 6. Prove that : $\int_0^1 \frac{dx}{e^x + e^{-x}} = \tan^{-1} e - \frac{\pi}{4}$

Q. 7. Prove that : $\int_0^{\pi} |\cos x| dx = 2$

Q. 8. Prove that : $\int_{-2}^2 \frac{dx}{1 + |x-1|} = log\, 2$

Q. 9. Prove that : $\int_{\pi/3}^{\pi/2} \frac{\sqrt{1+\cos x}}{(1-\cos x)^{5/2}} dx = \frac{3}{2}$

Q. 10. If $\int_0^1 \left(\frac{e^t}{1+t}\right) dt = a$ then show that $\int_0^1 \frac{e^t}{(1+t)^2} dt = a + 1 - \frac{e}{2}$

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10. Differential Equation

***NCERT* (0.1)**

Q. 1. Find the order and degree of the differential equation $y'' + 2y' + \sin y' = 0$.

$Ans: order = 2, degree = undefined$

Q. 2. Find the number of arbitrary constants in the particular solution of a differential equation of third order. $Ans: 0$

Q. 3. Find the number of arbitrary constants in the general solution of a differential equation of fourth order. $Ans: 4$

Q. 4. Solve the differential equation $(x + y)\frac{dy}{dx} = 1$ $Ans: y = \log(x + y + 1) + C$

Q. 5. Show that, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is the solution of the differential equation $xyy_2 + xy_1{}^2 - yy_1 = 0$

Q. 6. Show that $y = 3e^{2x} + 2e^{3x}$ is the solution of the differential equation $y_2 - 5y_1 + 6y = 0$

Q. 7. Show that $x^2 = 2y^2.\log y$ is solution of the differential equation $(x^2 + y^2)\frac{dy}{dx} - xy = 0$

Q. 8. Show that $y = x\sin x$, is the solution of the differential equation $xy_1 = y + x\sqrt{x^2 - y^2}$

Q. 9. Find the sum of order and degree of the differential equations, $(y''')^2 + (y'')^3 + (y')^4 + y^5 = 0$

Q. 10. Find the order and degree of the differential equation of family of curves $y = a\sin(x + b)$

$Ans: order = 2, degree = 1$

Q. 11. Show that $y = e^x(a\cos x + b\sin x)$ is the solution of the differential equation $y_2 - 2y_1 + 2y = 0$

Q. 12. Solve the differential equation $\frac{dy}{dx} = (1 + x^2 + y^2 + x^2y^2)$ $Ans: y = \tan\left(\frac{x^3}{3} + x + C\right)$

Q. 13. Form the differential equation representing the family of curves $y = a.\sin(x + b)$; $a, b \in R$.

$Ans: \frac{d^2y}{dx^2} + y = 0$

Q. 14. Find the integrating factor of the differential equation, $\frac{dy}{dx} - 3y\cot x = \sin 2x$. $Ans: cosec^3 x$

Q. 15. Form the differential equation of the family of circles touching the x-axis at origin. $Ans: \frac{dy}{dx} = \frac{2xy}{x^2 - y^2}$

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***HOTS* (0.2)**

Q. 1. Find the order and degree of the differential equation of family of parabola having axis as $x - axis$ and vertex at some point of $x - axis$. $Ans: order = 2, degree = 1$

Q. 2. Find the order and degree of the differential equation of family of curves $y = ae^{x+b} + ce^{x-d}$

$Ans: order = 1, degree = 1$

Q. 3. Find the sum of the order and the degree of the differential equations, $y_1 + \sqrt[3]{y_2} - x = 0$ $Ans: 2$

Q. 4. Find order and degree of the differential equation $y_2 + 3y_1{}^2 = x^2\log y_1$

$Ans: order = 2, degree = undefined$

Q. 5. Find the sum of order & degree of the differential equation $\sqrt[3]{y_2{}^3 + y_1{}^2} = \sqrt[2]{y_2{}^2 + y_1{}^3}$ $Ans: 6$

Q. 6. Show that $y = 3\cos(\log x) + 4\sin(\log x)$ is the solution of the differential equation $x^2y_2 + xy_1 + y = 0$

Q. 7. Solve the differential equation $(e^{2x} + 1)dy = (e^{2x} - 1)dx$ $Ans: y = \log|e^x + e^{-x}| + C$

Q. 8. Solve the differential equation $\sin 2x\, dy = \sqrt{\tan x}\, dx$ $Ans: y = \sqrt{\tan x} + C$

Q. 9. Solve the differential equation $(1 - \cot x)\, dy = dx$ $Ans: y = \frac{1}{2}\{x - \log|\sin x - \cos x|\} + C$

Q. 10. Solve the differential equation $\frac{dy}{dx} = (4x + y + 1)^2$ $Ans: 4x + y + 1 = 2\tan(2x + C)$

Q. 11. Solve the differential equations, $\sin^{-1}(y') = x + y$ $Ans: \tan(x + y) - \sec(x + y) = x + C$

Q. 12. Solve the differential equations, $\log(y') = 3x + 4y$. $Ans: 3e^{-4y} + 4e^{3x} = C$

Q. 13. Solve the differential equations, $\frac{dy}{dx} = \frac{\cos x}{1 - \cos x}$. $Ans: y = -\cot\left(\frac{x}{2}\right) - x + C$

Q. 14. Solve the differential equations, $\frac{dy}{dx} = \frac{x^4+1}{x^2-1}$. $Ans: y = \frac{x^3}{3} + x + 2log\left|\frac{x-1}{x+1}\right| + C$

Q. 15. Solve the differential equations, $\frac{dy}{dx} = \frac{2e^x+3e^{-x}}{3e^x+4e^{-x}}$

$Ans: y = \frac{1}{3}\log|4+3e^{2x}| - \frac{3}{8}\log|3+4e^{-2x}| + C$

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11. Vectors

***NCERT* (0.1)**

Q. 1. If a vector $\vec{r}$ is inclined to $x - axis$ at 45° and $y - axis$ at 60° if $|\vec{r}| = 8$, find the vector $\vec{r}$. $Ans: 4(\sqrt{2}i + j \pm k)$

Q. 2. If the vertices of a triangle ABC are $A(1,2,3), B(-1,0,0)$ & $C(0,1,2)$, then find $\angle ABC$. $Ans: cos^{-1}\left(\frac{10}{\sqrt{102}}\right)$

Q. 3. Find area of the triangle having vertices $(1,2,3), (0,3,5)$ and $(2,3,0)$. $Ans: \frac{\sqrt{30}}{2}$

Q. 4. Write down the all possible unit vectors in $YZ-plane$. $Ans: \frac{1}{\sqrt{y^2+z^2}}(yj+zk)\ ;\ cos\,\theta j + sin\,\theta k$

Q. 5. Find the projection & projection vector of $i + 2j - k$ on the vector $2i + 3j - 6k$ $Ans: \frac{12}{7}(2i+3j-6k)$

Q. 6. The two adjacent sides of a parallelogram are $2i-4j+5k$ & $i-2j-3k$. Find the unit vectors parallel to the diagonals. $Ans: \frac{1}{7}(3i-6j+2k)\ ;\ \frac{1}{\sqrt{69}}(i-2j+8k)$

Q. 7. $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, find the value of $(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a})$. $Ans: -\frac{3}{2}$

Q. 8. Find the direction of resultant of the vectors $\vec{a} = i-2j+k, \vec{b} = -2i+4j+5k$ & $\vec{c} = i-6j-7k$. $Ans: \frac{1}{\sqrt{17}}(-4j-k)$

Q. 9. Find the angle between two unit vectors if their resultant is again unit vector. $Ans: \frac{2\pi}{3}$

Q. 10. Find a & b if $(2i+6j+27k) \times (i+aj+bk) = \vec{0}$ $Ans: a = 3, b = \frac{27}{2}$

Q. 11. For the two vectors $\vec{a}$ & $\vec{b}$ prove that $|\vec{a} + \vec{b}| \le |\vec{a}| + |\vec{b}|$

Q. 12. For the vectors $\vec{a}, \vec{b}$ and $\vec{c}$. Show that $\vec{a} \times (\vec{b} \pm \vec{c}) = \vec{a} \times \vec{b} \pm \vec{a} \times \vec{c}$

Q. 13. If the points $(-1,-1,2), (2,m,5)$ & $(3,11,6)$ are collinear, find the value of m. $Ans: 8$

Q. 14. For the two vectors $\vec{a}$ & $\vec{b}$ prove that $|\vec{a}.\vec{b}| \le |\vec{a}|.|\vec{b}|$

Q. 15. Find the vector of magnitude 3 which is equally inclined to the coordinate axes. $Ans: \sqrt{3}(i+j+k)$

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***HOTS* (0.2)**

Q. 1. Find the value of $i.(j \times k) + j.(k \times i) + k.(j \times i)$ $Ans: 1$

Q. 2. If $\vec{a}$ is a unit vector then find the value of $|\vec{a} \times i|^2 + |\vec{a} \times j|^2 + |\vec{a} \times k|^2$ $Ans: 2|\vec{a}|^2$

Q. 3. If the vectors $\vec{a}$ and $\vec{b}$ be such that $|\vec{a}| = 3$ and $|\vec{b}| = \frac{\sqrt{2}}{3}$, and $\vec{a} \times \vec{b}$ is a unit vector. Find $|\vec{a} - \vec{b}|$ $Ans: \frac{65}{9}$

Q. 4. Prove that the points A, B & C with position vectors $\vec{a}, \vec{b}$ & $\vec{c}$ respectively are collinear if and only if $\vec{b} \times \vec{c} + \vec{c} \times \vec{a} + \vec{a} \times \vec{b} = \vec{0}$

Q. 5. If $\vec{a}$ & $\vec{b}$ are the position vectors of A and B, respectively, find the position vector of a point C in BA produced such that $BC = 1.5BA$. $Ans: \frac{3\vec{a} - \vec{b}}{2}$

Q. 6. If $|\vec{a} + \vec{b}| = 60$, $|\vec{a} - \vec{b}| = 40$ $|\vec{b}| = 46$ Find $|\vec{a}|$ $Ans: 22$

Q. 7. Find $|\vec{a} - \vec{b}|$, if two vectors $\vec{a}$ and $\vec{b}$ are such $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $\vec{a}.\vec{b} = 4$. *Ans*: $\sqrt{5}$

Q. 8. If $\overrightarrow{AB} = i + j + k$ & $\overrightarrow{BC} = 3i + j + 5k$, then find length of median through A in triangle ABC.

Ans: $\frac{1}{2}(5i + 3j + 7k)$

Q. 9. If three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ satisfies $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, prove that $\vec{b} \times \vec{c} = \vec{c} \times \vec{a} = \vec{a} \times \vec{b}$

Q. 10. If θ is angle between the two unit vectors $\vec{a}$ & $\vec{b}$, then prove that $sin\left(\frac{\theta}{2}\right) = \frac{|\vec{a} - \vec{b}|}{2}$

Q. 11. Using vector prove that, in any triangle ABC, $a = b.cos\,C + c.cos\,B$

Q. 12. If D & E are mid points of sides AB & AC of triangle ABC respectively, show that $\overrightarrow{BE} + \overrightarrow{DC} = \frac{3}{2}\overrightarrow{BC}$

Q. 13. Using vector prove that, in any triangle ABC, $cos\,A = \frac{b^2 + c^2 - a^2}{2bc}$

Q. 14. If $ABCD$ is quadrilateral and E and F are mid points of AC and BD respectively, show that $\overrightarrow{AB} + \overrightarrow{AD} + \overrightarrow{CB} + \overrightarrow{CD} = 4\overrightarrow{EF}$

Q. 15. If a line makes angles α, β & γ with coordinate axes, find the value of $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$.

Ans : -1

Q. 16. Show that the length of perpendicular drawn from a point having position vector $\vec{a}$ to a line joining position vectors $\vec{b}$ & $\vec{c}$ is $\frac{|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|}{|\vec{b} - \vec{c}|}$

Q. 17. If ABCDEF is a regular hexagon with centre O. Prove that $\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} = 6\overrightarrow{AO}$

Q. 18. Find the value of a for which the vector $(a^2 - 4)i + 2j - (a^2 - 9)k$ makes acute angles with the Co – ordinate axes. *Ans*: $(-3 \;\; -2) \cup (2 \;\; 3)$

Q. 19. If ABCDEF is a regular hexagon, then prove that $\overrightarrow{AD} + \overrightarrow{EB} + \overrightarrow{EC} = 4\overrightarrow{AB}$

Q. 20. If $\vec{a}, \vec{b}$ & $\vec{c}$ are non – zero and non collinear vectors. If $\vec{a} + 2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b} + 3\vec{c}$ is collinear with $\vec{a}$, then prove that $\vec{a} + 2\vec{b} + 6\vec{c} = \vec{0}$

Q. 21. If $3i + j + 4k$ & $i - j + k$ are the vector along the adjacent sides of a parallelogram. Find the acute angle between the diagonals of the parallelogram. *Ans*: $\frac{23}{\sqrt{697}}$

Q. 22. If $3\vec{a} + 4\vec{b} = \vec{c}$ and $\vec{a} - 3\vec{b} = 2\vec{c}$, show that $\vec{a}$ & $\vec{c}$ have same direction and $|\vec{a}| < |\vec{c}|$

Q. 23. If, $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}, \vec{a} \neq \vec{0}$ & $\vec{b} \neq \vec{c}$, show that $\vec{b} = \vec{c} + \lambda\vec{a}$ for some scalar λ.

Q. 24. If $\vec{a}, \vec{b}, \vec{c}$ & $\vec{d}$ are position vectors of four points such that $3\vec{a} - \vec{b} + 2\vec{c} - 4\vec{d} = \vec{0}$, show that the four points are coplanar.

Q. 25. If $\vec{a} \times \vec{b} = 2i + j - k$ and $\vec{a} + \vec{b} = i - j + k$ find the least value of $|\vec{a}|$. *Ans*: $\sqrt{2}$

Q. 26. Find the unit vector forming right handed orthogonal system with the perpendicular vectors $i + j$ & $i - j$. *Ans*: $\pm k$

Q. 27. If $|\vec{r}| = 3$, and $-1 \leq k \leq 3$ then find the interval in which $|k\vec{r}|$ lies. *Ans*: $[0 \;\; 9]$

Q. 28. Using vectors, prove that $cos(A - B) = cosA.\,cosB + sinA.\,sinB$.

Q. 29. If $\vec{a}$ & $\vec{b}$ are unit vectors at an angle of 30°. Find the area of the parallelogram having $\vec{d_1} = \vec{a} + 2\vec{b}$ and $\vec{d_2} = 2\vec{a} + \vec{b}$ as its diagonals. *Ans*: $\frac{3}{4}$ *sq.units.*

Q. 30. If $\vec{a}$ is a unit vector such that $\vec{a} \times i = j$, find $\vec{a}.i$ *Ans*: 0

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12. Three – Dimensions

NCERT (0.1)

Q. 1. Write down the vector form of the line $\frac{2x-2}{6} = \frac{3-y}{-4} = \frac{3z-4}{6}$

$Ans: \vec{r} = \left(i + 3j + \frac{4}{3}k\right) + \lambda(3i + 4j + 2k)$

Q. 2. Find the angle between the line joining the origin to the point $(2,1,1)$ and the line determined by the points $(3,5,-1)$ & $(4,3,-1)$. $Ans: \frac{\pi}{2}$

Q. 3. Find the point where the line through the points $A(3,4,1)$ & $B(5,1,6)$ crosses $XY-plane$.

$Ans: \left(\frac{13}{5}, \frac{23}{5}, 0\right)$

Q. 4. Find vector and Cartesian equations of the line joining the points $(3,-2,-5)$ & $(3,-2,6)$.

$Ans: \vec{r} = (3i - 2j - 5k) + \lambda k \,;\, \frac{x-3}{0} = \frac{y+2}{0} = \frac{z+5}{1}$

Q. 5. Find k, if the lines, $\frac{x-3}{-3} = \frac{y+2}{2k} = \frac{z+5}{2}; \frac{x-3}{3k} = \frac{y+2}{1} = \frac{z+5}{5}$ are perpendicular. $Ans: \frac{10}{7}$

Q. 6. Find the angle between the pair of lines, $\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}; \frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$ $Ans: \cos^{-1}\left(\frac{8\sqrt{3}}{15}\right)$

Q. 7. Show that the points $(2,3,4)$, $(-1,-2,1)$ & $(5,8,7)$ are collinear.

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HOTS (0.2)

Q. 1. Find the vector equation of the line $\frac{x-1}{2} = \frac{y-3}{-3}; z = 2$ $Ans: \vec{r} = (i + 3j + 2k) + \lambda(2i - 3j)$

Q. 2. Find vector equation of the line $x = 2\mu + 1 ; y = 2 ; z = 3$ $Ans: \vec{r} = (i + 2j + 3k) + 2\mu i$

Q. 3. Find the angle between any two diagonals of a cube. $Ans: cos^{-1}\left(\frac{1}{3}\right)$

Q. 4. If L, M & N are foot of perpendiculars drawn from $P(1,2,-3)$ on coordinate axes respectively. Find the equation of plane through the points L, M & N. $Ans: 6x + 3y - 2z = 6$

Q. 5. Find the vector equation of the line passing through the point $(2,0,5)$ and parallel to the line $6x - 2 = 3y + 1 = 2z - 2$. $Ans: \vec{r} = (2i + 5k) + \lambda(i + 2j + 3k)$

Q. 6. The x –coordinate of a point R on the line joining the points $P(2,2,1)$ and $Q(5,1,-2)$ is 4. Find the coordinates of point R. $Ans: \left(4, \frac{4}{3}, -1\right)$

Q. 7. Prove that the lines $x = py + q, z = ry + s$ & $x = p'y + q', z = r'y + s'$ are perpendicular if $pp' + rr' + 1 = 0$

Q. 8. If O is origin $OP = 3$ with $d.c's$ proportional to $-1, 2, -2$ then find the coordinates of P.

$Ans: (-1, 2, -2)$

Q. 9. If lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}; \frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then the value of k is $Ans: \frac{9}{2}$

Q.10. If a line makes angles α, β, γ with the positive directions of the coordinate axes, then find the value of $sin^2\alpha + sin^2\beta + sin^2\gamma$ $Ans: 2$

Q. 11. Find the point where the line through the points $A(3,4,1)$ & $B(5,1,6)$ crosses $x - axis$.

$Ans: No\ Point$

Q. 12. Find the point where the line through the points $A(3,1,2)$ & $B(5,3,6)$ crosses $x - axis$.

$Ans: (2,0,0)$

Q. 13. Show that the lines $\frac{x-1}{2} = \frac{y-1}{3} = \frac{z-1}{1}; \frac{x-3}{4} = \frac{y-2}{6} = \frac{z-1}{2}$ are parallel.

Q. 14. Show that the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{1}; \frac{x-3}{4} = \frac{y-2}{6} = \frac{z-3}{2}$ are coincident / identical.

Q. 15. P is a point on the line segment joining the points $(3,2,-1)$ & $(6,2,-2)$. If x co-ordinate of P is 5, then find its y co-ordinate. $Ans: 2$

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13. Probability

***NCERT* (0.1)**

Q. 1. If E and F are two independent events, prove that (i) E and F^l are also independent. (ii) E^l and F^l are also independent .

Q. 2. Given that the events A and B are such that $P(A) = \frac{1}{2}, P(A \cup B) = \frac{3}{5}$ & $P(B) = p$. Find p if A and B are (i) mutually exclusive (ii) independent $Ans: (i)\ \frac{1}{10} (ii) \frac{1}{5}$

Q. 3. A fair die is rolled. Consider events $E = \{1,3,5\}, F = \{2,3\}$ & $G = \{2,3,4,5\}$. Find $(i) P\left(\frac{E \cup F}{G}\right)$ $(ii) P\left(\frac{E \cap F}{G}\right)$ $Ans: (i)\ \frac{3}{4} (ii) \frac{1}{4}$

Q. 4. A die is tossed thrice. Find the probability of getting an odd number at least once. $Ans: \frac{7}{8}$

Q. 5. If A and B are two independent events, then the probability of occurrence of at least one of A and B is given by $1 - P(A')P(B')$

Q. 6. The mean of the numbers obtained on throwing a die having written 1 on three faces, 2 on two faces and 5 on one face. $Ans: 2$

Q. 7. A card is drawn from ten cards numbered 1 to 10. Find the probability that the card is of even number, if the number is more than 3? $Ans: \frac{4}{7}$

Q. 8. Given that the two numbers appearing on throwing two dice are different. Find the probability of the event 'the sum of numbers on the dice is 4'. $Ans: \frac{1}{15}$

Q. 9. If the two independent events A & B are such that $P(A) = 0.3, P(B) = 0.6$. Find $P(B\ alone)$ $Ans: 0.42$

Q. 10. If A & B are two events such that $P(A) = \frac{1}{4}, P(B) = \frac{1}{2}$ & $P(A \cap B) = \frac{1}{8}$. Find $P(not\ A\ \&\ not\ B)$. $Ans: \frac{3}{8}$

Q. 11. Assume that each born child is equally likely to be a boy or a girl, if a family has three children, find the probability that the eldest child is a girl if the family has at least one girl. $Ans: \frac{2}{3}$

Q. 12. Find the probability distribution of the number of successes in two tosses of a die, where a success is defined as six appears on at least one die.

Ans:

X	0	1
P(X)	$25/36$	$11/36$

Q. 13. A black and a red dice are rolled. Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5. $Ans: \frac{1}{3}$

Q. 14. An electronic assembly consists of two subsystems, say, A & B. From previous testing procedures, the following probabilities are assumed to be known: $P(A\ fails) = 0.2, P(B\ fails\ alone) = 0.15$, $P(A\ and\ B\ fail) = 0.15$ Evaluate the following (i) $P\left(\frac{A\ fails}{B\ has\ failed}\right)$ (ii) $P(A\ fails\ alone)$

$Ans: (i)\ \frac{1}{2} (ii) \frac{1}{20}$

Q. 15. If each element of a second order determinant is either zero or one, what is the probability that the value of the determinant is positive? $Ans: \frac{3}{16}$

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***HOTS* (0.2)**

Q. 1. The probability that a teacher will give a surprise test during any class is 0.2. If a student is absent thrice, what is the probability he will miss exactly one test? $Ans: 0.384$

Q. 2. Find the probability of getting at least one white ball, if 4 balls drawn with replacement from a bag containing 5 white, 7 red and 8 black balls. $Ans: 0.684$

Q. 3. A bag contains 3 white and 2 red balls, another bag contains 4 white and 3 red balls. One ball is drawn at random from each bag. Find the probability that the balls drawn are one white and one red.

$Ans: \frac{17}{35}$

Q. 4. Prove that the two events A & B are independent if, $P(\bar{A} \cap \bar{B}) = P(\bar{A})P(\bar{B})$.

Q. 5. A bag contains 30 tickets, numbered from 1 to 30. Five tickets are drawn at random and arranged in ascending order. Find the probability that the third number is 20. $Ans: \frac{855}{15834}$

Q. 6. Find the probability of obtaining an even prime number at least one die, when a pair of dice is rolled.

$Ans: \frac{11}{36}$

Q. 7. A committee of 4 students is selected at random from a group consisting 8 boys & 4 girls. Given that there is at least one girl on the committee, calculate the probability that there are exactly 2 girls on the committee. $Ans: \frac{168}{425}$

Q. 8. If 10% of the bulbs produced in a factory are of red colour, 20% are defective and2% are red and defective. If one bulb is picked up at random, find the probability of its being defective if it is red.

$Ans: \frac{1}{5}$

Q. 9. Let A & B be two events such that $P(A) = 0.6, P(B) = 0.2,$ & $P\left(\frac{A}{B}\right) = 0.5$. Then find $P\left(\frac{A'}{B'}\right)$.

$Ans: \frac{3}{8}$

Q. 10. A soldier fires three bullets on enemy, the probability that the enemy is killed by one bullet is 0.7. Find the probability that the enemy is killed. $Ans: 0.973$

Q. 11. Let A & B be two events such that $P(A) = \frac{1}{2}, P(B) = \frac{7}{12}$ & $P(not\ A\ or\ not\ B) = \frac{1}{4}$, then show that A & B are dependent.

Q. 12. A bag contains 5 red and 3 blue balls. If 3 balls are drawn, then find the probability that exactly two of the three balls were red, the first ball being red.

Q. 13. Let A & B be two events such that $P(A) = \frac{3}{10}, P(B) = \frac{2}{5}$ & $P(A \cup B) = \frac{3}{5}$, then find $P\left(\frac{A}{B}\right) + P\left(\frac{B}{A}\right)$

$Ans: \frac{7}{12}$

Q. 14. If A & B are mutually exclusive and exhaustive events and $3P(A) = P(B)$, then find value of $P(A)$

$Ans: 0.25$

Q. 15. The probability that at least one of the two events A & B occur is 0.6. If A & B occurs simultaneously with probability 0.3. Find $P(A') + P(B')$. $Ans: 1.1$

Q. 16. If A, B & C are three events then show that $P\left(\frac{A \cup B}{C}\right) = P\left(\frac{A}{C}\right) + P\left(\frac{B}{C}\right) - P\left(\frac{A \cap B}{C}\right)$

Q. 17. The probability that a certain person will buy a shirt is 0.2, the probability that he will buy a trouser is 0.3, and the probability that he will buy a shirt given that he buys a trouser is 0.4. Find the probability that he will buy a trouser given that he buys a shirt. $Ans: 0.6$

Q. 18. The probability of winning of two horses are $\frac{1}{3}$ & $\frac{1}{6}$ respectively. What is the probability at least one will win the race, when horses are running in (i) same race & (ii) different race. $Ans: (i)\frac{1}{2}\ (ii)\frac{4}{9}$

Q. 19. A speaks truth in 70% of the cases and B in 80% of the cases. In what percentage of the cases are they likely to contradict each other in stating the same fact? $Ans: 0.38$

Q. 20. A pair of die is rolled, if the sum of outcomes is an odd number, what is the probability that it is a prime? $Ans: \frac{7}{9}$

Q. 21. If A, B & C are three independent events such that $P(A) = P(B) = P(C) = p$, then find the probability that at least two of A, B, C occur. $Ans: 3p^2 - 2p^3$

Q. 22. A die is loaded, the probabilities of outcomes are: $P(1) = P(2) = 0.2, P(3) = P(5) = P(6) = 0.1$ & $P(4) = 0.3$. The die is thrown two times. Let A and B be the events, 'same number each time', and 'a total score is 10 or more', respectively. Determine whether or not A and B are independent.

$Ans: Independent$

Q. 23. Find the probability that the gun hits the plane. If an anti – aircraft gun can take a maximum of 4 shots at an enemy plane moving away from it. The probabilities of hitting the plane at first, second, third and fourth shot are $0.4, 0.3, 0.2$ & 0.1 respectively. $Ans: 0.6976$

Q. 24. Find the probability that the sum of the number obtained is neither a multiple of 2 nor a multiple of 3, if a pair of die is rolled. $Ans: \frac{1}{3}$

Q. 25. If the probability that at least one of the two events A and B occurs is p, and the probability that exactly one of the two events A and B occurs is q, then prove that $P(\bar{A}) + P(\bar{B}) = 2(1 - p) + q$

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