PUSHUP – 5 | Mathematics – XII | Max Marks – 80
Units : 12 + 13 + 1 | | Time – 3 hr

2 – Marks

$Q.1.$ Consider a function $f:\left[0,\ \frac{\pi}{2}\right] \to R$ given by $f(x) = sinx$ and $g:\left[0,\ \frac{\pi}{2}\right] \to R$ given by $g(x) = cosx$
Show that f and g are one – one but $f + g$ is not one – one.

$Q.2.$ Let $A = \{-1,0,1,2\}, B = \{-4,-2,0,2\}$ and $f, g : A \to B$ be functions defined by
$f(x) = x^2 - x, x \in A$ and $g(x) = 2\left|x - \frac{1}{2}\right| - 1, x \in A$. Show that f and g are equal.

$Q.3.$ Given that the events A and B are such that $P(A) = \frac{1}{2}, P(A \cup B) = \frac{3}{5}$ & $P(B) = p$. Find p if A and B are (i) mutually exclusive (ii) independent

$Q.4.$ If E and F are two independent events, prove that E and F^l are also independent

$Q.5.$ The corner points of the feasible region determined by the following system of linear inequalities:
$2x + y \le 10, x + 3y \le 15, x, y \ge 0$ are $(0,0), (5,0), (3,4)$ & $(0,5)$. Let $Z = px + qy$, where $p, q > 0$. Find the condition on p & q so that the maximum of Z occurs at both $(3,4)$ & $(0,5)$.

3 – Marks

$Q.6.$ If R_1 and R_2 are two equivalence relations, then show that $R_1 \cap R_2$ is also an equivalence relation.

$Q.7.$ Let N be the set of all natural numbers and R be the relation on $N \times N$ defined by :
$(a,b)R(c,d) \Leftrightarrow ad = bc$. Show that relation R is an equivalence relation.

$Q.8.$ Check whether the relation R defined over real numbers as $R = \{(a,b): a \le b^3\}$ is reflexive, symmetric or transitive.

$Q.9.$ Show that $f: N \to N$, given by $f(x) = \begin{cases} x+1\ ; x \text{ is odd} \\ x-1\ ; x \text{ is even} \end{cases}$ is both one-one and onto.

$Q.10.$ Let $f: N \to N$, given by $f(n) = \begin{cases} \frac{n+1}{2}\ ; n \text{ is odd} \\ \frac{n}{2}\ ; n \text{ is even} \end{cases}$ for all $n \in N$. State whether f is injective and surjective. Justify your answer.

$Q.11.$ Show that the function $f: R_+ \longrightarrow [-5\ \infty)$ defined by $f(x) = 9x^2 + 6x - 5$ is bijective.

$Q.12.$ Show that the function $f: R - \{2\} \longrightarrow R - \{3\}$ defined by $f(x) = \frac{3x+1}{x-2}$ is bijective.

$Q.13.$ Solve the linear programming problem graphically:
$Maximise\ Z = 5x + 3y$, subjected to $3x + 5y \le 15, 5x + 2y \le 10, x \ge 0, y \ge 0$

$Q.14.$ Solve the linear programming problem graphically:
$Minimise\ Z = 3x + 9y$, subjected to $x + 3y \le 60, x + y \ge 10, x \le y, x \ge 0, y \ge 0$

$Q.15.$ Solve the linear programming problem graphically:
$Maximise\ Z = x + y$, subjected to $x - y \le -1, y \le x, x \ge 0, y \ge 0$

$Q.16.$ Solve the linear programming problem graphically:
$Minimise\ \&\ Maximise\ Z = 5x + 10y$, subjected to $x + 2y \le 120, x + y \ge 60, x \ge 2y, x \ge 0, y \ge 0.$

$Q.17.$ A & B throw a die alternatively till one of them gets a 6 and wins the game. Find their respective probabilities of winning, if A starts first.

$Q.18.$ A coin is biased so that the head is three times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails.

$Q.19.$ In a game, a man wins a rupee for a six and loses a rupee for any other number when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amount he wins / loses.

$Q.20.$ Consider the experiment of tossing a coin. If the coin shows head, toss it again but if it shows tail, then throw a die. Find the conditional probability of the event that 'the die shows a number greater than 4' given that 'there is at least one tail'.

$5 - Marks$

$Q.21.$ Solve the linear programming problem graphically:

$Maximise\ Z = -x + 2y$, subjected to the constraints: $x \geq 3, x + y \geq 5, x + 2y \geq 6, y \geq 0$.

$Q.22.$ Show that the relation R in the set $A = \{x \in Z : 0 \leq x \leq 12\}$ given by $R = \{(a, b): |a - b|\ is\ a\ multiple\ of\ 4\}$ is an equivalence relation. Define equivalence class and evaluate $[1]$ and $[7]$

$Q.23.$ A man is to known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six .

$Q.24.$ A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both diamonds. Find the probability of the lost card being a diamond.

$Q.25.$ Assume that the chance of a patient having a heart attack is 40%. It is also assumed that a meditation and yoga course reduce the risk of heart attack by 30% and prescription of certain drug reduces its chances by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga?

******