<u>*PUSHUP – 4*</u> | <u>*Units : 9 + 10 + 11*</u>

<u>*Mathematics – XII*</u>

<u>*Max Marks – 80*</u> | <u>*Time – 3 hr*</u>

<u>$2 - Marks$</u>

$Q.1.$ Show that $y = c_1 e^{ax}\cos bx + c_2 e^{ax}\sin bx$; $c_1, c_2 \in real$ is the solution of the differential equation $y_2 - 2ay_1 + (a^2 + b^2)y = 0.$

$Q.2.$ If the vertices of a triangle ABC are $A(1,2,3), B(-1,0,0)$ & $C(0,1,2)$, then find $\angle ABC$

$Q.3.$ Find area of the parallelogram whose diagonals are $3i + j - 2k$, $i - 3j + 4k$

$Q.4.$ Prove that if a plane has the intercepts a, b, c and is at a distance of p units from the origin, then $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{p^2}$

$Q.5.$ If $P(1,2,-3)$ be the foot of perpendicular drawn from origin on the plane, then find the equation of the plane.

<u>$3 - Marks$</u>

$Q.6.$ Solve the differential equation: $x\frac{dy}{dx} + y - x + xy\cot x = 0$

$Q.7.$ Find the particular solution of the differential equation: $(x - y)(dx + dy) = dx - dy$; $x = 0, y = -1$

$Q.8.$ Show that the general solution of the differential equation $\frac{dy}{dx} + \frac{y^2 + y + 1}{x^2 + x + 1} = 0$ is given by $(x + y + 1) = A(1 - x - y - 2xy)$

$Q.9.$ Solve the initial value problem: $x^2 dy + (y^2 + xy)dx = 0; y(1) = 1$

$Q.10.$ If a unit vector $\vec{a}$ makes angles $\frac{\pi}{3}$ with i, $\frac{\pi}{4}$ with j and an acute angle θ with k , then find θ and hence the vector $\vec{a}$.

$Q.11.$ Let $\vec{a} = i + 4j + 2k, \vec{b} = 3i - 2j + 7k, \vec{c} = 2i - j + 4k$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, & $\vec{c}.\vec{d} = 15.$

$Q.12.$ Let $\vec{\alpha} = 3i - j, \vec{\beta} = 2i + j - 3k$, then express $\vec{\beta}$ in the form of $\vec{\beta} = \vec{\beta_1} + \vec{\beta_2}$ where $\vec{\beta_1}$ is parallel to $\vec{\alpha}$ and $\vec{\beta_2}$ is perpendicular to $\vec{\alpha}$.

$Q.13.$ If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitude, find the angle made by the vector $\vec{a} + \vec{b} + \vec{c}$ with $\vec{a}, \vec{b}$ & $\vec{c}$ respectively. Hence show that the vector $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to $\vec{a}$, $\vec{b}$ and $\vec{c}$.

$Q.14.$ Let $\vec{a}$ and $\vec{b}$ are two non – zero vectors**.** Then prove that $\vec{a}$ and $\vec{b}$ are perpendicular if and only if $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$

$Q.15.$ A girl walks $4km$ towards west, and then she walks $3km$ in a direction 30° east of north and stops. Determine the girl's displacement from her initial point of departure.

$Q.16.$ Find the value of λ so that the scalar product of the vector $i + j + k$ with the unit vector along the sum of vectors $2i + 4j - 5k$ & $\lambda i + 2j + 3k$ is equal to one.

$Q.17.$ Find the distance between the lines : $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$; $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$

$Q.18.$ Find the vector equation of the line passing through $(1,2,3)$ and parallel to the planes $\vec{r}.(i - j + 2k) = 5$ and $\vec{r}.(3i + j + k) = 6.$

$Q.19.$ If the points $(1,1,p)$ and $(-3,0,1)$ be equidistant from the plane $\vec{r}.(3i + 4j - 12k) + 13 = 0$, then find the value of p.

$Q.20.$ Find the vector and Cartesian equation of the plane passing through the line of intersection of the planes $\vec{r}.(i + j + k) = 1$ and $\vec{r}.(2i + 3j - k) + 4 = 0$ and parallel to $x-axis.$

<u>***P.T.O***</u>

Aniket's Mathematics

$5-$ *Marks*

$Q.21.$ Solve the initial value problem: $\frac{dy}{dx} + y\cot x = 2x + x^2 \cot x$; $y\left(\frac{\pi}{2}\right) = 0.$

$Q.22.$ Solve the differential equation: $(x^3 - 3xy^2)dx = (y^3 - 3x^2y)dy$

$Q.23.$ Find the distance of the point $(-1, -5, -10)$ from the point of intersection of the plane $\vec{r}.(i - j + k) = 5$ and the line $\vec{r} = 2i - j + 2k + \lambda(3i + 4j + 2k)$.

$Q.24.$ Show that the lines $\frac{x-a+d}{\alpha-\delta} = \frac{y-a}{\alpha} = \frac{z-a-d}{\alpha+\delta}$ and $\frac{x-b+c}{\beta-\gamma} = \frac{y-b}{\beta} = \frac{z-b-c}{\beta+\gamma}$ are coplanar, also find the equation of plane containing both the lines.

$Q.25.$ A line makes angles α, β, γ *and* δ with the four diagonals of a cube, prove that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$

******

Aniket's Mathematics