

2 – Marks

- Q. 1. If $y = \cos^{-1}x$, then find $\frac{d^2y}{dx^2}$ in terms of y alone.
- Q. 2. If $x = \sqrt{a^{\sin^{-1}t}}$ & $y = \sqrt{a^{\cos^{-1}t}}$ then prove that $\frac{dy}{dx} = -\frac{y}{x}$
- Q. 3. If $y = 3\cos(\log x) + 4\sin(\log x)$, prove that, $x^2y_2 + xy_1 + y = 0$
- Q. 4. If $y = 2\sqrt{\cot(x^2)}$, prove that $\frac{dy}{dx} = \frac{-2\sqrt{2}x}{\sin(x^2)\sqrt{\sin(2x^2)}}$
- Q. 5. If $x = a(\cos t + t\sin t)$ & $y = a(\sin t - t\cos t)$; find $\frac{d^2y}{dx^2}$

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- Q. 6. Test the continuity of the function $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & ; x \neq 0 \\ 0 & ; x = 0 \end{cases}$
- Q. 7. For what values of λ , the function $f(x) = \begin{cases} \lambda(x^2 - 2x) & ; x \leq 0 \\ 4x + 1 & ; x > 0 \end{cases}$ is continuous at $x = 0$
- Q. 8. Prove that the function defined by $f(x) = |x - 1|$; $x \in \text{Real}$, is not differentiable at $x = 1$.
- Q. 9. If $(x - a)^2 + (y - b)^2 = c^2$; $c > 0$, then prove that $\frac{(1 + (y_1)^2)^{3/2}}{y_2}$ is a constant free from a and b .
- Q. 10. If, $x\sqrt{1+y} + y\sqrt{1+x} = 0$; $-1 < x < 1$, then prove that $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$
- Q. 11. If, $y = \sin^{-1}x + \sin^{-1}\sqrt{1-x^2}$, $-1 < x < 1$. find $\frac{dy}{dx}$.
- Q. 12. If $y = e^{a \cos^{-1}x}$; $-1 < x < 1$, show that, $(1 - x^2)y_2 - xy_1 - a^2y = 0$.
- Q. 13. If $y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$, $x = \frac{\sin^3 t}{\sqrt{\cos 2t}}$, prove that $\frac{dy}{dx} = -\cot 3t$
- Q. 14. If $x^y + y^x + x^x = a^b$ then find $\frac{dy}{dx}$
- Q. 15. If $y = x^{\sin x} + (\sin x)^{\cos x}$ then find $\frac{dy}{dx}$
- Q. 16. Find the equation of normal to the curve $x^2 = 4y$ which passes through the point $(1, 2)$
- Q. 17. Show that $y = \log(1+x) - \frac{2x}{2+x}$, $x > -1$; is an increasing function of x throughout its domain.
- Q. 18. Find the intervals in which the function $f(x) = \frac{4\sin x - 2x - x \cos x}{2 + \cos x}$, is (i) increasing (ii) decreasing
- Q. 19. Find the intervals in which the function $f(x) = \log(\cos x)$, is (i) increasing (ii) decreasing
- Q. 20. Find the intervals in which the function $f(x) = (x + 1)^3(x - 3)^3$, is (i) increasing (ii) decreasing

5 – Marks

- Q. 21. Find the equation of tangent to the curve $y = \cos(x + y)$; $x \in [-2\pi, 2\pi]$, that are parallel to the line $x + 2y = 0$.
- Q. 22. Find the point on the curve $9y^2 = x^3$, where the normal to the curve makes equal intercepts with the axes.
- Q. 23. The sum of the perimeter of a circle and a square is k unit, where k is some constant. Prove that the sum of their areas is least when the side of square is double the radius of the circle.
- Q. 24. Show that the semi-vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$
- Q. 25. Find the greatest volume of the cylinder which can be inscribed in a right circular cone of height h and semi-vertical angle α .
