BACKUP – 1A **Mathematics – XII** **Max Marks – 80**

Time – 3 hr

$5 - Marks$

(1) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & -3 \\ -3 & 2 & -4 \end{bmatrix}$, find A^{-1}.

Hence solve the following system of equations: $x + 2y - 3z = -4$, $2x + 3y + 2z = 2$, $3x - 3y - 4z = 11$

(2) A school wants to award its students for the values of Honesty, Regularity and Hard work with a total cash award d of Rs. 6000. Three times the award money for Hard Work added to that given for Honesty amounts to Rs 11000. The award money given for Honesty and Hard work together is double the one given for Regularity. Represent the above situation algebraically and find the award money for value, using matrix method. Apart from these values, namely Honesty, Hard work and Regularity, suggest one more value which the school must include for award.

(3) An apache helicopter of enemy is flying along the curve $y = x^2 + 7$. A soldier placed at $(3, 7)$ want to hit down the helicopter when it is at the minimum distance. Find the minimum distance.

(4) A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is $10m$. Find the dimensions of the window to admit maximum light through the whole opening.

(5) The sum of the perimeter of a circle and a square is k unit, where k is some constant. Prove that the sum of their areas is least when the side of square is double the radius of the circle.

(6) Show that the relation R in the set $A = \{x \in Z : 0 \le x \le 12\}$ given by
$R = \{(a, b) : |a - b| \text{ is a multiple of } 4\}$ is an equivalence relation. Define equivalence class and evaluate $[1]$ and $[7]$

(7) Find the equation of tangent to the curve $y = cos(x + y)$; $x \in [-2\pi\ \ 2\pi]$, that are parallel to the line $x + 2y = 0$.

$3 - Marks$

(8) Show that the function $f: R_+ \to [-5\ \ \infty)$ defined by $f(x) = 9x^2 + 6x - 5$ is bijective

(9) Let $f: N \to N$, given by $f(n) = \begin{cases} \frac{n+1}{2} & ; n \text{ is odd} \\ \frac{n}{2} & ; n \text{ is even} \end{cases}$ for all $n \in N$. State whether f is injective and surjective. Justify your answer.

(10) Let N be the set of all natural numbers and R be the relation on $N \times N$ defined by :
$(a, b)R(c, d) \Leftrightarrow ad = bc$. Show that relation R is an equivalence relation.

(11) Simplify : $tan^{-1}\left\{\frac{\sqrt{1 + x^2} - 1}{x}\right\}$; $|x| \neq 0$

(12) Simplify : $tan^{-1}\left\{\frac{cos\,x}{1 - sin\,x}\right\}$; $-\frac{\pi}{2} < x < \frac{\pi}{2}$

(13) Simplify : $cos\left\{sin^{-1}\left(\frac{3}{5}\right) - sin^{-1}\left(\frac{8}{17}\right)\right\}$

(14) Find x if, $sin^{-1}(1 - x) - 2sin^{-1}(x) = \frac{\pi}{2}$

(15) Show that: $tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}cos^{-1}x; x\in[0\ \ 1]$

(16) If, $A=\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}$, then by using $P.M.I$, show that $A^n=\begin{bmatrix}1+2n & -4n\\ n & 1-2n\end{bmatrix}\ \forall\ n\in \mathrm{N}$.

(17) Find matrix D if $A=\begin{bmatrix}2 & -1\\ 3 & 4\end{bmatrix}$, $B=\begin{bmatrix}5 & 2\\ 7 & 4\end{bmatrix}$, $C=\begin{bmatrix}2 & 5\\ 3 & 5\end{bmatrix}$ if $CD-AB=O$

(18) Test the continuity of the function $f(x)=\begin{cases}x^2 sin\left(\frac{1}{x}\right) & ; x\neq 0\\ 0 & ; x=0\end{cases}$

(19) Prove that the function defined by $f(x)=|x-1|$; $x\in Real$, is not differentiable at $x=1$.

(20) If $(x-a)^2+(y-b)^2=c^2$; $c>0$, then prove that $\frac{\left(1+(y_1)^2\right)^{3/2}}{y_2}$ is a constant free from a and b.

(21) If, $x\sqrt{1+y}+y\sqrt{1+x}=0$; $-1<x<1$, then prove that $\frac{dy}{dx}=-\frac{1}{(1+x)^2}$

(22) Find $\frac{dy}{dx}$ if, $y=x^{sin\ x}+\{sin\ x\}^{cos\ x}$.

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