TEST PAPER – 2 (D. A. V. Special)

Mathematics – XI

Time : 3 hr **Max Marks : 100**

General Instructions :

1. All questions are compulsory.

2. The question paper consists of **29 questions** divided into three sections **A, B** and **C.** **Section A** comprises of **10** questions of **one mark** each, **Section B** comprises of **12** questions of **four marks** each and **Section C** comprises of **07** questions of **six marks** each.

3. All questions in **Section A** are to be answered in one word, one sentence or as per the exact requirement of the question.

4. There is no overall choice. However, internal choice has been provided in **04** questions of **four marks** each and **02** questions of **six marks** each. You have to attempt only one of the alternatives in all such questions.

5. Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION – A

(1) Let $f, g : \mathbf{R} \to \mathbf{R}$ be defined, respectively by $f(x) = x + 1$, $g(x) = 2x - 3$. Find $\frac{f}{g}$

(2) Express the complex number $\left(i^{18} + i^{-25}\right)^3$, in the form of $a + ib$.

(3) If $y = \frac{x^3}{\cos^n x}$, find $\frac{dy}{dx}$

(4) Evaluate : $\lim_{x \to 0} (\operatorname{cosec} x - \cot x)$

(5) Find the intercept made by the line $2x + 3y + 6 = 0$ on the coordinate axes.

(6) Find the components of the statement ***'p'***
p: *A mixture of alcohol and water can be separated by chemical methods.*

(7) Write the contrapositive of the statements **"***If x is a prime number, then x is odd* **".**

(8) Write the converse of the statements ***'t '***
t* :** *If the two lines are parallel, then they do not intersect in the same plane.**

(9) There are four men and six women on the city council**.** If two council member is selected for a committee at random**,** how likely is it that it is a man and a woman**?**

(10) A letter is chosen at random from the word **'*ASSASSINATION*'.** Find the probability that letter is a vowel.

P.T.O

Section – B

(11) For any sets **A** and **B,** show that **P (A ∩ B) = P (A) ∩ P (B).**

(12) Let **R** be a relation from **N** to **N** defined by **R = {(a, b) : a, b ∈ N** and **a = b**^{2} **}.** Are the following true?
(i) (a, a) ∈ R, for all **a∈N** **(ii) (a, b) ∈ R, ⇒ (b, a) ∈ R** **(iii) (a, b) ∈ R, (b, c) ∈ R ⇒ (a, c) ∈ R.**

(13) Show that : $\dfrac{\sec 8x - 1}{\sec 4x - 1} = \dfrac{\tan 8x}{\tan 2x}$ **OR** Prove that : $\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+2\cos 16\theta}}}} = 2\cos\theta$

(14) In a triangle **ABC** prove that **:** $\tan\left(\dfrac{A-B}{2}\right) = \dfrac{a-b}{a+b}\cot\left(\dfrac{C}{2}\right)$

(15) What is the number of ways of choosing **4** cards from a pack of **52** playing cards? In how many of these
(i) four cards are of the same suit, **(ii)** four cards belong to four different suits.

OR

Find the number of words with or without meaning which can be made using all the letters of the word **AGAIN.** If these words are written as in a dictionary, what will be the **50th** word**?**

(16) Prove that **:** $Re\left(\dfrac{1}{1-\cos x + 2i\sin x}\right) = \dfrac{1}{5+3\cos x}$

(17) Find the sum of the first ***n*** terms of the series**: 3 + 7 +13 +21 +31 + - - - .**

(18) Find the equation of the line through the point **(3 , 2)** and which makes an angle **45^o** with **x – 2y = 3.**

OR

Find the equation of the line passing through the point **(2 , 2)** and cutting off intercepts on axes whose sum is **9.**

(19) Find the coordinates of foci and vertices**,** the eccentricity and the length of latus rectum of the hyperbola $9y^2 - 4x^2 = 36$ **.**

OR

Find the equation of conic – section such that**,** $e = \dfrac{3}{4}$ **,** foci on **y – axis,** centre at origin and passing through the point **(6 , 4).**

(20) A point **R** with x-coordinate **4** lies on the line segment joining the points **P(2, –3, 4)** and **Q (8, 0, 10).** Find the coordinates of the point **R.**

(21) Using first principle**,** find the derivative of the function $f(x) = x^2 . \sin x$

OR

Find the value of **'*a*'** and **'*b*',** so that $\lim_{x \to 1} f(x) = f(1)$, for the function

$$f(x) = \begin{cases} 5ax - 2b & ; x < 1 \\ 11 & ; x = 1 \\ 3ax + b & ; x > 1 \end{cases}$$

(22) Out of **100** students**,** two sections of **40** and **60** are formed**.** If you and your friend are among **100** students**.** What is the probability that**,**
(i) you both enter the same section **?** ***(ii)*** you both enter the different section **?**

Section – C

(23) ***(i)*** A solution of **8%** boric acid is to be diluted by adding a **2%** boric acid solution to it . The resulting mixture is to be more than **4%** but less than **6%** boric acid**.** If we have **640** litres of the **8%** solution, how many litres of the **2%** solution will have to be added?

(ii) A man wants to cut three lengths from a single piece of board of length **91cm .**The second length is to be **3cm** longer than the shortest and the third length is to be twice as long as the shortest. What are the possible lengths of the shortest board if the third piece is to be at least **5cm** longer than the second ?

(24) Find the general solution of the equation : $\tan x + \tan\left\{x + \frac{\pi}{3}\right\} + \tan\left\{x + \frac{2\pi}{3}\right\} = 3$

(25) There are **200** individuals with a skin disorder**, 120** had been exposed to the chemical **C_1, 50** to chemical **C_2,** and **30** to both the chemicals **C_1** and **C_2.** Find the number of individuals exposed to

(i) Chemical **C_1** but not chemical **C_2** ***(ii)*** Chemical **C_1** or chemical **C_2 .** ***(iii)*** No Chemical

(26) Using principle of mathematical induction prove that**,**

for all $n \geq 1$, $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \text{- - -} + n \text{ terms} = \frac{n(n+1)^2(n+2)}{12}$

(27) Find ***a***, ***b*** and ***n*** in the expansion of $(a + b)^n$ if the first three terms of the expansion are **729, 7290** and **30375,** respectively.

OR

(i) Show that the coefficient of the middle term in the expansion of $(1 + x)^{2n}$ is equal to the sum of the coefficients of two middle terms in the expansion of $(1 + x)^{2n-1}$.

(ii) If in the expansion of $(1 + x)^n$, the coefficient of 5^{th}, 6^{th} and 7^{th} terms are in **A.P.** Find ***n***.

(28) Let **S** be the sum**, P** the product and **R** the sum of reciprocals of n terms in a **G.P.** Prove that $P^2 R^n = S^n$.

OR

If the p^{th}, q^{th} and r^{th} terms of an ***A.P.*** as well as a ***G. P.*** are ***a, b*** and ***c*** respectively**.**

Prove that**:** $a^{b-c} . b^{c-a} . c^{a-b} = 1$.

(29) Find the mean and standard deviation using short-cut method.

Classes	**0 – 30**	**30 – 60**	**60 – 90**	**90 – 120**	**120 –150**	**150 –180**	**180 -210**
Frequency	**2**	**3**	**5**	**10**	**3**	**5**	**2**

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