ANIKET ' S

OPERATION

MATHEMATICS

CBSE (XI) – 2023

LEVEL - II

(13^{th} Revised Edition)

DATE : 1^{st} JULY 2022

PREFACE

It is known to every student that ***70 – 80 %*** *questions in* ***CBSE – XI*** *Exam have been asked from* ***NCERT*** *Text Book, and the remaining* ***20 – 30 %*** *are* ***HOTS*** *(**H**igh **O**rder **T**hinking **S**kills) questions.*

After going through the market available books on Mathematics for ***CBSE XI****, I come to know that all books contains a lot of* ***HOTS*** *questions which are* ***Out of Course*** *for* ***CBSE XI,*** *and students are wasting their valuable time on such questions.*

As an outcome, in spite of Hard Labour students had performed below the level, what they expect from themselves, in their ***Unit Tests*** *or* ***Formative Assessment Examinations****.*

*So, this '**Operation Mathematics CBSE XI – 2023 (Level – II)'** has been designed to provide some selective questions of* ***HOTS*** *for the target oriented preparation of* ***CBSE XI.***

This package will not only bring confidence, but help the students in scoring the wonder tons ***90 – 100 % Marks*** *in coming* ***CBSE XI – 2023 Exam.***

Aniket Manohar

Mob : 90–134–135–43,

1. Set Theory

Q. 1. If $X = \{8^n - 7n - 1 : n \in N\}$ and $Y = \{49(n-1) : n \in N\}$, Then prove that $X \subseteq Y$.

Q. 2. For any two sets A and B, prove that $(i) A \cup B = A \cap B \Leftrightarrow A = B$. $(ii) (A - B) \cup B = A \Leftrightarrow B \subset A$.

Q. 3. Let A, B & C be three sets such that $A \cup B = C$ and $A \cap B = \phi$. Then prove that $A = C - B$.

Q. 4. In an examination, 80% students passed in Mathematics, 72% passed in Science and 13% failed in both subjects. If 312 students passed in both subjects find the total number of students who appeared in the examination. $Ans: 480$

Q. 5. In a certain town 25% families own a phone, 15% own a car, 65% own neither a phone nor a car. If 2000 families own both the phone and the car find how many families (i) live in the town? (ii) own either a phone or a car? $Ans: (i) 40{,}000\ (ii) 14{,}000$

Q. 6. In a group of children, 35 play football out of which 20 play football only, 22 play hockey, 25 play cricket out of which 11 play cricket only. Out of these 7 play cricket and football but not hockey, 3 play football and hockey but not cricket and 12 play football and cricket both. How many play (i) all the three games? (ii) cricket and hockey but not football? (ii) hockey only (iv) at least one of the tree games? $Ans: (i)\ 5\ (ii)\ 2\ (iii)\ 12\ (iv)\ 60.$

Q. 7. In certain locality of a town of 10,000 families, it was found that 40% families buy newspaper A, 20% families buy newspaper B and 10% families buy newspaper C. 5% families buy A & B, 3% families buy B & C and 4% families buy A & C. If 2% families buy all the three newspaper, find the number of families which buy. (i) A only (ii) B only (iii) None of A, B & C $Ans: (i) 3300\ (ii) 1400\ (iii) 4000$

Q. 8. The report of one survey of 100 students stated that the number of student studying the various language were : Sanskrit, Hindi and Punjabi – 5 ; Hindi and Sanskrit – 10 ; Punjabi and Sanskrit – 8 ; Hindi and Punjabi – 20; Sanskrit – 30; Punjabi – 50; Hindi – 23. The surveyor who prepared this report was fired. Why? $Ans: n(OH) < 0$

Q. 9. A class has 175 students, the number of students studying the various subjects in the class is: Mathematics 100, Physics 70, Chemistry 46, Mathematics & Physics 30, Mathematics & Chemistry 28, Physics & Chemistry 23, Mathematics, Physics & Chemistry 18. How many students are enrolled in (i) Exactly one subject (ii) Exactly two subject (iii) None of the three subject. $Ans: (i) 108 (ii) 27 (iii) 22$

Q. 10. In a survey of 100 students, the number of students studying the various languages was found to be, English only 18, English but not Hindi 23, English and Sanskrit 8, English 26, Sanskrit 48, Sanskrit & Hindi 8, no language 24. Find how many students were studying (i) Hindi? (ii) English and Hindi? $Ans: (i) 18\ (ii) 3$

Q. 11. A survey shows that 63% of Indians like coffee, whereas 76% likes tea. If x% of Indians like both coffee and tea, find the range of possible values of x. $Ans: 39 \le x \le 63.$

Q. 12. From 50 students taking examinations in Mathematics, Physics and Chemistry, each of the student has passed in at least one of the subject, 37 passed Mathematics, 24 Physics and 43 Chemistry. At most 19 passed Mathematics and Physics, at most 29 Mathematics and Chemistry and at most 20 Physics and Chemistry. What is the largest possible number that could have passed all three examinations? $Ans: 14$

Q. 13. Two finite sets have m and n elements respectively. The total number of subsets of first set is 56 more than the total number of subsets of the second set. Find the values of m and n. $Ans: m = 6, n = 3$

Q. 14. Write the sets in set builder form $(i) \{5, 11, 19, 29, 41, - - -\}$ $(ii) \{3, 7, 13, 21, 31, - - -\}$
$Ans: (i) \{x : x = n^2 + 3n + 1;\ \forall n \in N\}$ $(ii) \{x : x = n^2 + n + 1;\ \forall n \in N\}$

Q. 15. For any set A, $n(OA)$ = number of elements in the set A only. Prove that for three sets A, B & C,

$$n\big((OA) \cup (OB) \cup (OC)\big) = n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(B \cap C) - 2n(C \cap A) + 3n(A \cap B \cap C)$$

******

2. Relations and Functions

Q. 1. Find the domain and range of the following functions: $(i)\ \frac{1}{\sqrt{16 - x^2}}$ $(ii)\ \frac{x^2 + 2x + 1}{x^2 - 8x + 12}$

$Ans: (i) (-4, 4)\ ;\ [0.25, \infty)\ \ (iv) R - \{2, 6\}\ ;\ (-\infty, -3] \cup [0, \infty)$

Q. 2. If, $f = \{(0,1),(2,0),(3,-4),(4,2),(5,1)\}$ and $g = \{(1,0),(2,2),(3,-1),(4,4),(5,3)\}$ are any two functions, find the domain and range of the following functions $(i) f + g$ $(ii) f - g$ $(iii) f.g$ $(iv) \frac{f}{g}$

Ans: $domain$ (i) to (iv): $\{2,3,4,5\}$;

$range$: $(i)\{2,-5,6,4\}$ (ii) $\{-2,-3\}$ (iii) $\{0,4,8,3\}$ (iv) $\left\{0,4,\frac{1}{2},\frac{1}{3}\right\}$

Q. 3. Let R be a relation from real to real defined by $R = \{(a,b): a \le b^3\}$. Are the following true?
(i) $(a,a) \in R$, $for\ all\ real\ a$ (ii) $(a,b) \in R \Rightarrow (b,a) \in R$ (iii) $(a,b) \in R, (b,c) \in R \Rightarrow (a,c) \in R$.

Ans: (i) No (ii) No (iii) No

Q. 4. A relation R is a defined on Q(rational numbers) by, $R = \{(a,b): |a - b| \le \frac{1}{2}$. Are the following true?
(i) $(a,a) \in R$, $for\ all\ a \in Q$ (ii) $(a,b) \in R \Rightarrow (b,a) \in R$ (iii) $(a,b) \in R, (b,c) \in R \Rightarrow (a,c) \in R$.

Ans: (i) Yes (ii) Yes (iii) No

Q. 5. Let R be a relation defined by $R = \{(a,b): |a - b|\ divisible\ by\ n;\ a,b,n \in Z\}$. Are the following true?
(i) $(a,a) \in R$, $for\ all\ a \in Z$ (ii) $(a,b) \in R \Rightarrow (b,a) \in R$ (iii) $(a,b) \in R, (b,c) \in R \Rightarrow (a,c) \in R$.

Ans: (i) Yes (ii) Yes (iii) Yes

Q. 6. A relation R is defined as $R = \{(x,y): x + 2y = 41;\ x,y \in N\}$. Find domain, co-domain and range of R?

Ans : $Domain = \{1,3,5,---,39\}$; $Co\text{-}domain = N$; $Range = \{1,2,3,---,20\}$

Q. 7. Let $A = \{-1,0,1,2\}$; $B = \{-4,-2,0,2\}$ and $f,g: A \to B$ be functions defined by $f(x) = x^2 - x; \forall x \in A$ and $g(x) = 2\left|x - \frac{1}{2}\right| - 1$; $\forall x \in A$. Are f & g equal? Justify your answer. Ans: Yes

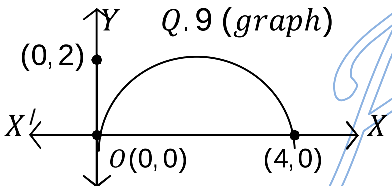
Q. 8. Let $f = \{(1,7),(2,12),(0,4),(-1,3)\}$ be a quadratic function on $Z(integers)$. Find $f(x)$.

Ans: $f(x) = x^2 + 2x + 4$

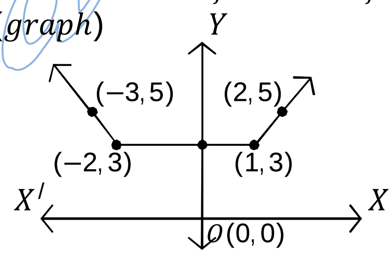
Q. 9. Find the domain and range of the functions $f(x) = \sqrt{4x - x^2}$ Ans: $D_f = [0,4], R_f = [0\ \ 2]$

Q. 10. Find the domain and range of the functions $f(x) = |x - 1| + |x + 2|$ Ans: $D_f = Real, R_f = [3,\infty)$

$Q.9$ $(graph)$



$Q.10$ $(graph)$



******

3. Trigonometry

Prove the following ($Q.1 - Q.12$)

Q. 1. $sin^3 x + sin^3\left(x + \frac{2\pi}{3}\right) + sin^3\left(x + \frac{4\pi}{3}\right) = -\frac{3}{4} sin\, 3x$

Q. 2. $sin\, 20^\circ . sin\, 40^\circ . sin\, 60^\circ . sin\, 80^\circ = \frac{3}{16}$

Q. 3. $cos\left(\frac{\pi}{5}\right) . cos\left(\frac{2\pi}{5}\right) . cos\left(\frac{4\pi}{5}\right) . cos\left(\frac{8\pi}{5}\right) = -\frac{1}{16}$

Q. 4. $tan\left(\frac{\pi}{24}\right) = \sqrt{6} - \sqrt{3} - \sqrt{4} + \sqrt{2}$

Q. 5. $cos\, 5x = 16\, cos^5 x - 20\, cos^3 x + 5\, cos\, x$

Q. 6. $\frac{sec\, 8x - 1}{sec\, 4x - 1} = \frac{tan\, 8x}{tan\, 2x}$

Q. 7. $\sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + 2\cos 16\theta}}}} = 2\cos\theta$

Q. 8. $cos\left(\frac{\pi}{10}\right) = \frac{\sqrt{10 + 2\sqrt{5}}}{4}$

Q. 9. $3(sinx - cosx)^4 + 4(sin^6 x + cos^6 x) + 6(sinx + cosx)^2 = 13$

Q. 10. $cos^4\left(\frac{\pi}{8}\right) + cos^4\left(\frac{3\pi}{8}\right) + cos^4\left(\frac{5\pi}{8}\right) + cos^4\left(\frac{7\pi}{8}\right) = \frac{3}{2}$

Q. 11. $\left\{1 + \cos\left(\frac{\pi}{8}\right)\right\}\left\{1 + \cos\left(\frac{3\pi}{8}\right)\right\}\left\{1 + \cos\left(\frac{5\pi}{8}\right)\right\}\left\{1 + \cos\left(\frac{7\pi}{8}\right)\right\} = \frac{1}{8}$

Q. 12. $2sin^2\beta + 4cos(\alpha + \beta).\sin\alpha \sin\beta + \cos 2(\alpha + \beta) = \cos 2\alpha$

Q. 13. If α and β are two different roots of the equation, $a\cos\theta + b\sin\theta = c$, then prove that

(i) $sin\,(\alpha + \beta) = \frac{2ab}{a^2 + b^2}$ (ii) $tan\,(\alpha + \beta) = \frac{2ab}{a^2 - b^2}$ (iii) $cos\,(\alpha + \beta) = \frac{a^2 - b^2}{a^2 + b^2}$

Q. 14. If, $\sin A + \sin B = a, \cos A + \cos B = b$, then prove that,

$(i)\ \sin(A+B) = \frac{2ab}{a^2+b^2}$ $(ii)\ \cos(A+B) = \frac{b^2-a^2}{a^2+b^2}$ $(iii)\ \tan\left(\frac{A-B}{2}\right) = \frac{\sqrt{4-a^2-b^2}}{\sqrt{a^2+b^2}}$

Q. 15. If $\sin(\theta+\alpha) = a$ and $\sin(\theta+\beta) = b$, then prove that $\cos 2(\alpha-\beta) - 4ab\cos(\alpha-\beta) = 1-2a^2-2b^2$

Q. 16. Prove that: $\tan x + \tan\left(x+\frac{\pi}{3}\right) + \tan\left(x+\frac{2\pi}{3}\right) = 3\tan 3x$

Q. 17. If $x\cos\theta = y\cos\left(\theta+\frac{2\pi}{3}\right) = z\cos\left(\theta+\frac{4\pi}{3}\right)$, then find the value of $xy+yz+zx$. $Ans: 0$

Q. 18. Prove that : $\sqrt{3}\,cosec\, 20^\circ - \sec 20^\circ = 4$

Q. 19. If angle θ is divided into two parts such that the tangent of one part is k times the tangent of other, and $\emptyset$ is their difference, then show that $(k-1)\sin\theta = (k+1)\sin\emptyset$

Q. 20. Find the value of, $\sin\left(\frac{x}{2}\right), \cos\left(\frac{x}{2}\right)$ & $\tan\left(\frac{x}{2}\right)$ if, $\sin x = -\frac{1}{4}; \frac{3\pi}{2} \le x \le 2\pi$

$Ans: \frac{\sqrt{5}-\sqrt{3}}{2}, -\frac{\sqrt{5}+\sqrt{3}}{2}, \sqrt{15}-4$

******

5. Complex Number

Q. 1. If z_1, z_2, z_3 are complex numbers such that, $|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right| = 1$,

Then prove that $|z_1+z_2+z_3| = 1$

Q. 2. If $z = x+iy, z^{1/3} = a-ib$ and $bx-ay = kab(a^2-b^2)$, then find the value of k. $Ans: 4$

Q. 3. Prove that : $Re\left\{\frac{1}{1-\cos x+2i\sin x}\right\} = \frac{1}{5+3\cos x}$

Q. 4. If $a+ib = \frac{3}{2+\cos x+i\sin x}$, prove that $a^2+b^2 = 4a-3$

Q. 5. Solve for x: $x^2-(5+i)x+(18-i) = 0$. $Ans: 3+4i, 2-3i$

Q. 6. Evaluate : $(i)\ \sqrt{-7-24i}$ $(ii)\ \sqrt{8-15i}$ $Ans: (i) \pm(3-4i)\ (ii) \pm\left\{\frac{5}{\sqrt{2}}-\frac{3i}{\sqrt{2}}\right\}$

Q. 7. If $|z| = 1$, prove that $\frac{z-1}{z+1}$ is purely imaginary complex number.

Q. 8. If $|w| = 1$, and $w = \frac{1-zi}{z-i}$ then prove that z is purely real complex number.

Q. 9. If, $w = \frac{2z+1}{iz+1}$, such that $Im(w) = -2$, then show that the locus of the point representing z in the Argand plane is a straight line.

Q. 10. If, $x = 1+2i$, find the values of $(i)\ x^2-2x+5$ $(ii)\ x^3+7x^2-x+33$ $(iii)\ x^4-x^3-2x^2+2x+3$.

$Ans: (i) 0\ (ii) 24i\ (iii) 15-26i$

Q. 11. If a and b are any two complex number then prove that $|a+b|^2+|a-b|^2 = 2\{|a|^2+|b|^2\}$

Q. 12. If z_1 and z_2 be two complex numbers, prove that $|z_1+z_2| \le |z_1|+|z_2|$

Q. 13. If $|Z^2-1| = |Z|^2+1$, then show that the complex number Z lies on imaginary axis.

Q. 14. Find the value of P such that the difference of the roots of the equation $x^2-Px+8 = 0$ is 2.

Q. 15. Find the value of k if for the complex numbers Z_1 and Z_2 we have,

$|1-\overline{Z_1}Z_2|^2 - |Z_1-Z_2|^2 = k(1-|Z_1|^2)(1-|Z_2|^2)$

******

6. Inequations

Q. 1. Solve the inequation : $\frac{x-1}{2x+1} < \frac{x-3}{2x-3}$; $x \in R$ $Ans: \left(-\frac{1}{2}\quad \frac{3}{2}\right)$

Q. 2. Solve the inequation : $\frac{2x+4}{x-1} \ge 5$; $x \in R$ $Ans: (1\quad 3]$

Q. 3. Solve the inequation : $\left|\frac{3x-4}{2}\right| \leq \frac{5}{12}$; $x \in R$ Ans: $\left[\frac{19}{18} \quad \frac{29}{18}\right]$

Q. 4. Solve the inequation: $|2x-3| < |x+5|$; $x \in R$ Ans: $(-0.667 \quad 8)$

Q. 5. Solve the inequation : $|x-2| + |x-3| \geq 6$; $x \in R$ Ans: $(-\infty \quad -0.5] \cup [5.5 \quad \infty)$

Q. 6. Solve the inequation for integral x : $\frac{|x+3|+x}{x+2} > 1$ Ans: $(-5 \quad -2) \cup (-1 \quad \infty)$

Q. 7. A solution of 9% acid is to be diluted by adding 3% acid solution to it. The resulting mixture is to be more than 5% but less than 7% acid. If there is 460 liters of the 9% solution, how many liters of 3% solution will have to be added? Ans: $> 230l$ & $< 920l$

Q. 8. A plumber can be paid under two schemes given as, $Scheme - I$: $Rs\,600$ and $Rs.\,50$ per hour, $Scheme - II$: $Rs.\,170$ per hour. If the job takes n hours, for what values of n does the scheme I gives the plumber better wages? Ans: $n < 5$

Q. 9. The water acidity in a pool is considered normal when the average pH reading of three daily measurements is between 7.2 and 7.8. If the first two pH readings are 7.48 and 7.85, find the range of pH value for the third reading that will result in the acidity level being normal. Ans: < 8.07 & > 6.27

Q. 10. The cost function $C(x)$, and revenue function $R(x)$ for a product is given by $C = 400 + 2x$ and $R = 6x + 20$ respectively, where 'x' is the number of items produced and sold. How many items must be sold to realize some profit? Ans: $x > 95$

******

7. Permutation & Combination

Q. 1. Find the 50^{th} word if letters of the word $SUCCESS$ are arranged according to the dictionary.
Ans: $CSCUESS$

Q. 2. Find the summation of all possible four digit numbers formed by $1, 2, 3, 4$ & 5 with no digits repeated.
Ans: 399960

Q. 3. If the letters of the word $INDIAN$ are arranged according to a dictionary, find the rank of $INDIAN$.
Ans: 105

Q. 4. Find the total number of words with and without meaning can be formed from the letters of the word $MATHEMATICS$ taken four at a time. Ans: 2454

Q. 5. How many natural number not exceeding 4321 can be formed with the digits $1, 2, 3$ & 4, if the digits can be repeated? Ans: 313

Q. 6. Find the total number of ways of drawing 2 balls of different colour from a bag containing 2 white, 3 red, 5 green, 4 black balls. Ans: 71

Q. 7. A boy has 3 library tickets and 8 books of his interest in the library. Of these 8, he does not want to borrow Mathematics Part II, unless Mathematics Part I is also borrowed. In how many ways can he choose the three books to be borrowed? Ans: 41

Q. 8. Find the number of different 8 $letter$ arrangements that can be made from the letters of the word $DAUGHTER$ so that, (i) No two vowels are together. Ans: 14400
(ii) Respective position of consonants and vowels remains unchanged. Ans: 720

Q. 9. Four letters are dictated to four persons and an envelope is addressed to each of them, the letters are put into the envelopes at random so that each envelope contains exactly one letter. Find the total number of way so that (i) exactly one letter is in its proper envelope. (ii) No letter is in its proper envelope. (iii) Exactly two letters are in its proper envelope. Ans: (i) 8 (ii) 9 (iii) 6

Q. 10. On a New Year day each student in a class sends a card to each student. The postman delivers 600 cards. How many students are there in there in the class? Ans: 25

Q. 11. Find the number of diagonals that a polygon can have. Ans: $\frac{n(n-3)}{2}$

Q. 12. Out of 18 points in a plane, no three are in the same line except five points which are collinear. Find the number of (i) lines and (ii) triangles that can be formed joining the point. Ans: $(i)144$ $(ii)806$

Q. 13. There is a polygon of n sides $(n > 5)$. Triangles are formed by joining the vertices of the polygon. How many triangles are there? Also prove that the numbers of these triangles which have no side common with any of the sides of the polygon is $\frac{n(n-4)(n-5)}{6}$

Q. 14. In how many ways can the letters of the word $ARRANGE$ be arranged so that (i) the two $R's$ are never together. (ii) the two $A's$ are together but not the two $R's$. (iii) neither the two $A's$ nor the two $R's$ are together. $Ans: (i)\ 900\ (ii)\ 240\ (iii)\ 660$

Q. 15. Prove that, $C(2n,n) = {}^{2n}C_n = \frac{1.3.5.---(2n-1)}{n!}2^n$

******

8. Binomial Theorem

Q. 1. Find the value of k so that the term independent of x in $\left(\sqrt{x}-\frac{k}{\sqrt[3]{x}}\right)^{10}$ is 13440. $Ans: k = \pm 2$

Q. 2. If in the expansion of $(1+x)^n$, the coefficient of $14^{th}, 15^{th}\ \&\ 16^{th}$ terms are in $A.P$. Find n. $Ans: n = 23 or 34$

Q. 3. Using Binomial theorem, find remainder when $6^n - 5n - 9$ is divided by 25. $Ans: 17$

Q. 4. Prove that the middle term of the expansion $(1+4x+4x^2)^n$ is $\frac{1.3.5.7.---.(2n-1)4^n.\ x^n}{n!}$; $n \in Z_+$

Q. 5. Find the coefficient of x^5 in the expansion of $(1+x)^{21} + (1+x)^{22} + --- + (1+x)^{30}$ $Ans: {}^{31}C_6 - {}^{21}C_6$

Q. 6. Find the coefficient of x^4 in the expansion of $(1+x+x^2+x^3)^{11}$. $Ans: 990$

Q. 7. Find the coefficient of x^{-1} in the expansion of $(1+3x^2+x^4)\left(1+\frac{1}{x}\right)^8$ $Ans: 232$

Q. 8. Find a, b and n in the expansion of $(a+b)^n$ if the $3^{rd}, 4^{th}\ \&\ 5^{th}$ terms of the expansion are $84, 280$ and 560 respectively. $Ans: n = 7, a = 1, b = 2$

Q. 9. The coefficients of three consecutive terms in the expansion of $(1+a)^n$ are in the ratio $91:42:15$. Prove that $n = 18$.

Q. 10. If a_1, a_2, a_3 and a_4 are the coefficient of any four consecutive terms in the expansion of $(1+x)^n$, prove that, $\frac{a_1}{a_1+a_2} + \frac{a_3}{a_3+a_4} = \frac{2a_2}{a_2+a_3}$

Q. 11. If $3^{rd}, 4^{th}, 5^{th}\ \&\ 6^{th}$ terms in a binomial expansion are $a, b, c\ \&\ d$ respectively, prove that $\frac{b^2-ac}{c^2-bd} = \frac{5a}{3c}$

Q. 12. If a_1 , a_2 and a_3 are coefficient of any three consecutive terms in the expansion of $(1+x)^n$, prove that $n = \frac{2a_1a_3 + a_2(a_1+a_3)}{{a_2}^2 - a_1a_3}$

Q. 13. Prove that the coefficient of r^{th} term from end of the expansion $(a+x)^n$ is equals to the coefficient of r^{th} term of the expansion $(x+a)^n$.

Q. 14. If A be the sum of odd terms and B the sum of even terms in the expansion of $(a+x)^n$, prove that $A^2 - B^2 = (a^2 - x^2)^n$

Q. 15. Using Binomial theorem, find remainder when 5^{26} is divided by 13. $Ans: 12$

******

9. Sequence & Series

Q. 1. If the $p^{th}, q^{th}\ \&\ r^{th}$ terms of an $A.P$ as well as a $G.P$ are $a, b\ \&\ c$ respectively prove that $a^{b-c}.b^{c-a}.c^{a-b} = 1$

Q. 2. If $p\ \&\ q$ are the two arithmetic means and G be the geometric mean between the numbers $a\ \&\ b$, prove that, $G^2 = (2p-q)(2q-p)$

Q. 3. (i) If a, b, c, d are four distinct positive quantities in $A.P.$, then show that $bc > ad$
(ii) If a, b, c, d are four distinct positive quantities in $G.P.$, then show that $a + d > b + c$

Q. 4. If a, b, c are in $A.P$ prove that $\frac{1}{\sqrt{b}+\sqrt{c}}, \frac{1}{\sqrt{c}+\sqrt{a}}, \frac{1}{\sqrt{a}+\sqrt{b}}$ are also in $A.P$

Q. 5. Show that $(x^2+xy+y^2),(z^2+xz+x^2)$ & (y^2+yz+z^2) are consecutive terms of an $A.P.$, if x, y & z are in $A.P.$

Q. 6. Find the natural number a for which $\sum_{k=1}^{n} f(a+k) = 16(2^n - 1)$, where the function f satisfies $f(x+y) = f(x).f(y)$ for all natural numbers x, y and further $f(1) = 2$.

Q. 7. If the p^{th} and q^{th} terms of a $G.P.$ are q and p respectively, show that its $(p+q)^{th}$ term is $\left(\frac{q^p}{p^q}\right)^{\frac{1}{p-q}}$

Q. 8. If A is the arithmetic mean and G_1 & G_2 be two geometric means between any two numbers, then prove That $2A = \frac{{G_1}^2}{G_2} + \frac{{G_2}^2}{G_1}$

Q. 9. If $A_1, A_2, A_3 - - - A_n$ are in $A.P$ with common difference of d then prove that,

$$sec(A_1).\ sec(A_2) + sec(A_2).\ sec(A_3) + - - - + sec(A_{n-1}).\ sec(A_n) = \frac{tan(A_n) - tan(A_1)}{sin\ d}$$

Q. 10. If $a_1, a_2, a_3 - - - a_n$ are in $A.P$, where $a_i's > 0\ \forall\ i$, then prove that,

$$\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \frac{1}{\sqrt{a_3}+\sqrt{a_4}} + - - - + \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}} = \frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$$

Q. 11. Find the sum of first 24 terms of the $A.P.$ $a_1, a_2, a_3, \ldots$ if it is known that $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225.$ $Ans: 900$

Q. 12. If the sum of an infinite geometric series is 15 and the sum of the squares of these terms is 45, find the series. $Ans: 5, \frac{10}{3}, \frac{20}{9}, \frac{40}{27}, - - -$

Q. 13. The sum of first two terms of an infinite geometric series is 15 and each term is equal to the sum of all the terms following it. Find the series. $Ans: 10, 5, \frac{5}{2}, \frac{5}{4}, \frac{5}{8}, - - -$

Q. 14. Sum the series to infinity $x(x+y) + x^2(x^2+y^2) + x^3(x^3+y^3) + - - -$ with $|x| < 1$ & $|y| < 1$ $Ans: \frac{x^2+xy-2x^3y}{1-x^2-xy+x^3y}$

Q. 15. A square is drawn by joining the mid points of the sides of a given square. A third square is drawn inside the second square in the same way, and this process continues indefinitely. If a side of the first square is $16\ cm$, determine the sum of the areas of the squares. $Ans: 512\ cm^2$

******

10. Straight Lines

Q. 1. Two consecutive sides of a parallelogram are $4x + 5y = 0$ & $7x + 2y = 0$. If the equation of the one diagonal is $11x + 7y = 9$, find the equation of the other diagonal. $Ans: y = x$

Q. 2. If the slope of a line passing through the point $A(3, 2)$ is $\frac{3}{4}$, then find points on the line which are 5 units away from the point A. $Ans: (-1, -1), (7, 5)$

Q. 3. At white point the origin must be shifted so that the coefficients of x and y in the new equation obtained from $x^2 + y^2 + 2x + 4y = 2$ is 0. $Ans: (-1, -2)$

Q. 4. If p be the length of perpendicular drawn from origin on the line $\frac{x}{a} + \frac{y}{b} = 1$ such that a^2, p^2, b^2 are in $A.P$, prove that $a^4 + b^4 = 0$

Q. 5. A ray of light is sent along the line $x - 2y = 3$, upon reaching the line $3x - 2y = 5$, the ray is reflected from it. Find the equation of the line containing the reflected ray. $Ans: 29x - 2y = 31.$

Q. 6. Find the equation of the straight line passing through the point $(-2, -7)$ and having an intercept of length 3 between the straight lines $4x + 3y = 12$ & $4x + 3y = 3$. $Ans: x = -2, 7x + 24y + 182 = 0$

Q. 7. Find the coordinates of orthocenter of the triangle whose vertices are $(1, 2), (2, 3)$ & $(4, 3)$. $Ans: (1, 6)$

Q. 8. Two vertices of a triangle are $(3, -1)$ & $(-2, 3)$ and its orthocenter is at origin. Find the coordinates of the third vertex. $Ans: \left(-\frac{36}{7}, -\frac{45}{7}\right)$

Q. 9. Find the distance of the point $(2,3)$ from the line $2x-3y+9=0$ measured along the line $x-y+2=0$

$Ans: 4\sqrt{2}$

Q. 10. Two equal sides of an isosceles triangle are given by the equations $y=7x+3$ & $x+y=3$ and its third side passes through the point $(1,-10)$, find the equation of the third side.

$Ans: x-3y=31, 3x+y+7=0.$

Q. 11. Find the locus of the centers of circles touching the straight lines (equation of the angle bisectors of the angle between the lines) $3x-4y+7=0$ & $12x-5y=8$ $Ans: 77y=99x+51, 21x+27y=131$

Q. 12. One diagonal of a square lies along the line $x-2y+2=0$ and one vertex of the square is $(1,4)$. Find the equations of all the sides and the other diagonal of the square.

$Ans: sides: x+3y=13, x+3y=3, 3x-y+1=0, 3x-y=9 \; diagonal: 2x+y=6$

Q. 13. Find the equations of the legs of right angle isosceles triangle right angled at $(3,2)$ with hypotenuse is along the line $x-2y=3$. $Ans: 3x-y=7$ & $x+3y=9$

Q. 14. A variable line which always remains at a constant distance p from origin, cuts the coordinate axes at A, B respectively**.** Prove that the locus of the point of intersection of the lines drawn parallel to the coordinate axes through A and B is $\frac{1}{x^2}+\frac{1}{y^2}=\frac{1}{p^2}$**.**

Q. 15. A variable line which always remains at a constant distance $3p$ from origin, cuts the coordinate axes at A, B respectively**.** Prove that the locus of the centroid of the triangle OAB is $\frac{1}{x^2}+\frac{1}{y^2}=\frac{1}{p^2}$

******

11. Conic – Section

Q. 1. Show that the given points $(3,-2), (1,0), (-1,-2), and\ (1,-4)$ are concyclic.

Q. 2. Find the equation of the circumscribing circle of the triangle formed by the line $bx+ay=ab$ and coordinate axes. $Ans: x^2+y^2-ax-by=0$

Q. 3. Find the equation of the circle circumscribing the triangle formed by the lines $x+y=6, 2x+y=4$ and $x+2y=5$ $Ans: x^2+y^2-17x-19y+50=0$

Q. 4. Find the intercept on axes made by a circle having $(-4,3)$ and $(12,-1)$ as ends of a diameter.

$Ans: 4\sqrt{66}, 4\sqrt{13}$

Q. 5. Find the equation of the circle whose centre is $(3,-1)$ and which cut off an intercept of length 6 from the line $2x-5y+18=0$. $Ans: x^2+y^2-6x+2y-28=0$

Q. 6. If the lines $4x-3y+12=0$ and $3x+4y=16$ are tangents to a circle at the points $(-3,0)$ and $(4,1)$ respectively, find the equation of the circle. $Ans: x^2+y^2-2x+6y-15=0$

Q. 7. One of the diameter of the circle circumscribing the rectangle $ABCD$ is $4y=x+7$. If A and B are the points $(-3,4)$ and $(5,4)$ respectively, find area of the rectangle. $Ans: 32\ sq.units$

Q. 8. Find the vertex, focus, LLR, axis and directrix of the parabola $x^2+8x+12y+4=0$

$Ans: (-4,1); (-4,-3); 12; x+4=0; y=4.$

Q. 9. If y_1, y_2 & y_3 be the ordinates of vertices of the triangle inscribed in a parabola $y^2=4ax$, then show that the area of the triangle is $\frac{1}{8a}\left|(y_1-y_2)(y_2-y_3)(y_3-y_1)\right|$

Q. 10. Find the centre, length of axes, eccentricity, foci of the ellipse $25x^2+9y^2-150x-90y+225=0$.

$Ans: (3,5); \; a=10, b=6; \; e=\frac{4}{5}; \; foci: (3,1), (3,9)$

Q. 11. Find the centre, length of axes, eccentricity, foci of the hyperbola $x^2-2y^2-2x+8y-1=0$.

$Ans: (1,2); \; a=2\sqrt{3}, b=2\sqrt{6}; \; e=\sqrt{3}; \; foci: (1,5), (1,-1)$

Q. 12. The foci of a hyperbola coincide with foci of ellipse $9x^2+25y^2=225$. If eccentricity of the hyperbola is 2 find the equation of hyperbola. $Ans: 3x^2-y^2=12$

Q. 13. Find the equation of conic section such that, $e=\frac{3}{4}$, foci on $y-axis$, centre at origin and passing through the point $(6,4)$. $Ans: 16x^2+7y^2=688$

Q. 14. Find the equation of the conic whose focus, directrix and eccentricity are respectively $(-1,1)$, $x-y+3=0$ and $\frac{1}{2}$ $Ans: 7x^2+7y^2+2xy+10x-10y+7=0$

Q. 15. Find the eccentricity of an ellipse if its latus rectum is equal to half of its major axis, also find the equation of ellipse if centre is origin and vertices are $(0,\ \pm 5)$. $Ans: \frac{1}{\sqrt{2}}, 2x^2+y^2=25$

******

12. Three – Dimension

Q. 1. Find the distance of the point $(-1,3,4)$ from $x-axis$. $Ans: 5\ units$

Q. 2. Find the image of the point $(-1,3,4)$ in $x-axis$. $Ans: (-1,-3,-4)$

Q. 3. Find the coordinates of the point where the line through $(3,-4,-5)$ & $(2,-3,1)$ crosses the plane $2x+y+z=7$. $Ans: (1,-2,7)$

Q. 4. Find the point at which the line segment joining the points $(4,8,10)$ and $(6,10,8)$ crosses $YZ-plane$. $Ans: (0,4,14)$

Q. 5. Using section formula show that the points $(0,7,-10),(1,6,-6)$ & $(4,9,-6)$ are non collinear.

Q. 6. If $A(3,2,0), B(5,3,2), C(-9,6,-3)$ are the vertices of a triangle, find the length AD, if AD bisects the internal angle A of the triangle ABC. $Ans: \frac{13\sqrt{6}}{16}$

Q. 7. Show that the points $A(5,-1,1), B(7,-4,7), C(1,-6,10)$ & $D(-1,-3,4)$ are the vertices of a rhombus and not of a square.

Q. 8. Find the coordinates of centroid of triangle ABC if $(0,7,-10),(1,6,-6)$ & $(4,9,-6)$ are coordinates of middle points of the sides BC, CA & AB respectively. $Ans: \left(\frac{5}{3}, \frac{22}{3}, -\frac{22}{3}\right)$

Q. 9. Find the coordinates of centroid of triangle ABC if A, B & C are feet of perpendiculars drawn from the point $P(1,2,3)$ on coordinate planes respectively. $Ans: \left(\frac{2}{3}, \frac{4}{3}, 2\right)$

Q. 10. The mid-points of the sides of a triangle are $(5,7,11),(0,8,5)$ & $(2,3,-1)$. Find its vertices $Ans: (-3,4,-7),(7,2,5)\&(3,12,17)$

******

13. Limits & Derivatives

Q. 1. Find the value of a and b, so that $\lim_{x\to 1} f(x) = f(1)$, if, $f(x) = \begin{cases} 5ax-2b & ; x<1 \\ 11 & ; x=1 \\ 3ax+b & ; x>1 \end{cases}$ $Ans: a=3, b=2$

Q. 2. Find the value of a, so that $\lim_{x\to 0} f(x) = f(0)$, if $f(x) = \begin{cases} a\sin\left\{(x+1)\frac{\pi}{2}\right\} & ; x\le 0 \\ \frac{\tan x - \sin x}{x^3} & ; x>0 \end{cases}$ $Ans: a = \frac{1}{2}$

Q. 3. Find the value of a, so that $\lim_{x\to \frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$, if, $f(x) = \begin{cases} \frac{1-\sin^3 x}{3\cos^2 x} & ; x<\frac{\pi}{2} \\ a & ; x=\frac{\pi}{2} \\ \frac{b(1-\sin x)}{(\pi-2x)^2} & ; x>\frac{\pi}{2} \end{cases}$ $Ans: a=\frac{1}{2}, b=4$

Q. 4. Find value of k, so that $\lim_{x\to 0} f(x) = f(0)$, if $f(x) = \begin{cases} \frac{1-\cos 4x}{x^2} & ; x<0 \\ k & ; x=0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & ; x>0 \end{cases}$ $Ans: k=8$

Q. 5. Show that $\lim_{x\to 0} f(x)$ does not exist, for the function $f(x) = \begin{cases} \frac{e^{1/x}-1}{e^{1/x}+1} ; & x\ne 0 \\ k & ; x=0 \end{cases}$

Q. 6. Find value of a, b, c, so that $\lim_{x\to 0} f(x) = f(0)$, if $f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x} & ; x < 0 \\ c & ; x = 0 \\ \frac{\sqrt{x+bx^2} - \sqrt{x}}{b\,x^{3/2}} & ; x > 0 \end{cases}$

$Ans: a = -\frac{3}{2}, c = \frac{1}{2}, b = real\ number\ other\ than\ 0$

Q. 7. Prove that : $\lim_{x\to 0}\left\{\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt[3]{1+x} - \sqrt[3]{1-x}}\right\} = \frac{3}{2}$

Q. 8. Prove that: $\lim_{x\to 1}\left\{\frac{(2x-3)(\sqrt{x}-1)}{3x^2+3x-6}\right\} = -\frac{1}{2}$

Q. 9. Prove that: $\lim_{x\to 0}\left\{\frac{\sqrt{1+x^2} - \sqrt{1+x}}{\sqrt{1+x^3} - \sqrt{1+x}}\right\} = 1$

Q. 10. Prove that : $\lim_{x\to \frac{\pi}{4}}\left\{\frac{\sin x - \cos x}{x - \frac{\pi}{4}}\right\} = \sqrt{2}$

Q. 11. Prove that : $\lim_{x\to 0}\left\{\frac{\tan x - \sin x}{\sin^3 x}\right\} = \frac{1}{2}$

Q. 12. Prove that: $\lim_{x\to \frac{\pi}{4}}\left\{\frac{\tan^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}\right\} = -4$

Q. 13. Prove that : $\lim_{x\to 0}\left\{\frac{1 - \cos x\sqrt{\cos 2x}}{x^2}\right\} = \frac{3}{2}$

Q. 14. Prove that: $\lim_{x\to 0}\left\{\frac{x\left(e^{2+x} - e^2\right)}{1 - \cos x}\right\} = 2e^2$

Q. 15. Prove that: $\lim_{x\to 2}\left\{\frac{3^x + 3^{3-x} - 12}{3^{3-x} - 3^{x/2}}\right\} = -\frac{4}{3}$

Q. 16. Prove that: $\lim_{x\to 1}\left\{\frac{x^2 - \sqrt{x}}{\sqrt{x} - 1}\right\} = 3$

Q. 17. Prove that: $\lim_{x\to 0}\left\{\frac{(x+y)\sec(x+y) - x\sec x}{y}\right\} = \sec x(\tan x + 1)$

Q. 18. Prove that: $\lim_{x\to 0}\left\{\frac{(a+x)^2 \sin(a+x) - a^2 \sin a}{x}\right\} = 2a\sin a + a\cos^2 a$

Q. 19. Find derivative of the followings :

$(i) x^2.\sin x.\log x$ $(ii) x^3.e^x.\sin x$ $(iii) \frac{\sin x - x\cos x}{x\sin x + \cos x}$ $(iv) \frac{x^3.\ \sin x}{\cos x}$

$Ans: (i)\ 2x.\sin x.\log x + x^2.\cos x.\log x + x\sin x$ $(ii)\ x^2e^x(3\sin x + x\sin x + x\cos x)$

$(iii) \frac{x^2}{(x\sin x + \cos x)^2}$ $(iv)\ x^3\sec^2 x + 3x^2\tan x$

Q. 20. Find derivative of the followings using first principle (*ab – initio, limit process, delta process*):

$(i) \log(\sin 2x)$ $(ii) \frac{\sin x}{x}$ $(iii) \frac{\sqrt{x}}{\cos x}$ $(iv) e^{\sin x}$ $(v) \sqrt{x} + \frac{1}{\sqrt{x}}$

$Ans: (i)\ 2\cot 2x$ $(ii) \frac{x\cos x - \sin x}{x^2}$ $(iii) \frac{\sec x + 2x\sec x\tan x}{2\sqrt{x}}$ $(iv) \cos x.e^{\sin x}$ $(v) \frac{x-1}{2\,x\sqrt{x}}$

Q. 21. Find derivative of the followings using first principle (ab – initio) :

$(i)\sqrt{\sin x}$ $(ii) \tan\sqrt{x}$ $(iii) \sqrt[3]{\cot x}$ $(iv) \sin\sqrt[3]{x}$ $(v) cosec\,(ax+b)$

$Ans: (i) \frac{\cos x}{2\sqrt{\sin x}}$ $(ii) \frac{\sec^2\sqrt{x}}{2\sqrt{x}}$ $(iii) -\frac{cosec^2 x}{3(\cot x)^{2/3}}$ $(iv) \frac{\cos\sqrt[3]{x}}{3(x)^{2/3}}$ $(v) -a\,cosec\,(ax+b).\cot\,(ax+b)$

Q. 22. If, $y = \sqrt{\frac{x}{a}} + \sqrt{\frac{a}{x}}$, prove that, $2xy\frac{dy}{dx} = \frac{x}{a} - \frac{a}{x}$

Q. 23. If, $y = \sqrt{x} + \frac{1}{\sqrt{x}}$, prove that, $2x\frac{dy}{dx} + y = 2\sqrt{x}$

******

14. Probability

Q. 1. A card is drawn at random from a pack of 52 playing cards. Find the probability of getting a king or a heart or a red card. $Ans: \frac{7}{13}$

Q. 2. A and B throw a die alternatively till one of them gets a '6' and wins the game. Find their respective probabilities of winning, if A starts first. $Ans: \frac{6}{11}, \frac{5}{11}$

Q. 3. A bag contains 30 tickets, numbered from 1 to 30. Five tickets are drawn at random and arranged in ascending order. Find the probability that the third number is 20. $Ans: \frac{285}{5278}$

Q. 4. A speaks truth in 70% of the cases and B in 80% of the cases, in stating the same fact. Find the percentage of the cases ***(i)*** they likely to contradict each other. ***(ii)*** they likely to coincide each other.
$Ans: (i) 0.38\ (ii) 0.62$

Q. 5. Probability of solving specific problem independently by A & B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently , find the probability that
(i) the problem is solved (ii) exactly one of them solves the problem $Ans: (i)\frac{2}{3}\ (ii)\frac{1}{2}$

Q. 6. From the 21 tickets marked through 1 to 21. Three tickets are drawn at random. Find the probability that the numbers on the tickets drawn are in $A.P.$ $Ans: \frac{10}{133}$

Q. 7. Four Students A, B, C, D have given a question to solve. If A is twice as likely to solve the problem as B, and B and C are given about the same chance of solving the problem, while C is twice as likely to solve the problem as D, what is the probability that the question is solved. $Ans: 0.7$

Q. 8. Four letters are dictated to four persons and an envelope is addressed to each of them, the letters are inserted into the envelopes at random so that each envelope contains exactly one letter. Find the probability that. (i) Exactly one letter is in its proper envelope. (ii) Exactly two letters is in its proper envelope. (iii) No letter is in its proper envelope. (iv) All letters are not in its proper envelope.
$Ans: (i)\frac{1}{3}\ (ii)\ \frac{1}{4}\ (iii)\ \frac{9}{24}\ (iv)\frac{23}{24}$

Q. 9. A bag contains 4 red, 3 white & 5 blue balls. If three balls are drawn at random, determine the probability that all the three balls are of: (i) same colour. (ii) different colour $Ans: (i)\frac{3}{44}\ (ii)\frac{3}{11}$

Q. 10. In a $Bag\ I$ there are 5 white, 8 red balls, in $Bag\ II$ there are 7 white, 6 red balls and in $Bag\ III$ there are 6 white and 5 red balls. One ball is taken out at random from each bag. Find the probability that all three balls are of the same colour. $Ans: \frac{450}{1859}$

Q. 11. In a $Bag\ I$ there are 5 white, 4 red balls, in $Bag\ II$ there are 7 white, 2 red balls and in $Bag\ III$ there are 4 white and 5 red balls. One bag is selected and two balls are drawn at random from the bag. Find the probability that the balls drawn are red . $Ans: \frac{17}{108}$

Q. 12. Let the sample space $S = \{x_1, x_2, x_3, x_4, x_5, x_6\}$. If Respective probabilities are given as assignment in the table. Find $(i) k\ \ (ii) P(x_6)$

X	x_1	x_2	x_3	x_4	x_5	x_6
$P(X)$	k	$2k$	$2k$	$3k$	$3k^2$	$7k^2 + k$

$Ans: (i)\frac{1}{10}\ (ii)\frac{17}{100}$

Q. 13. If probabilities of an assignment are given as $P(X = x) = \begin{cases} k(x+1) & ; for\ x = 1,2,3,4 \\ 2kx & ; for\ x = 5,6,7 \\ 0 & ; otherwise \end{cases}$

Find $(i) k\ \ (ii) P(3 < x < 6)$ $Ans: (i)\frac{1}{50}\ (ii)\frac{3}{10}$

******