BACK – UP TEST – 1 B

Mathematics – XI

Max Marks – 100
Time – 3 hr

6 – Marks

Q. 1. The 3^{rd}, 4^{th} and 5^{th} terms in the binomial expansion $(x + a)^n$ are **84, 280** and **560,** respectively. Find **'*x*', '*a*'** and **'*n*'.**

Q. 2. Find the number of arrangements of the letters of the word ***INDEPENDENCE.*** In how many of these arrangements, **(*i*)** do the vowels never occur together **(*ii*)** do the words begin with a vowel?

Q. 3. If $a + ib = \dfrac{3}{2 + \cos x + i \sin x}$, prove that $a^2 + b^2 = 4a - 3$

Q. 4. If in a triangle **ABC,** $\dfrac{b+c}{12} = \dfrac{c+a}{13} = \dfrac{a+b}{15}$, prove that $\dfrac{\cos A}{2} = \dfrac{\cos B}{7} = \dfrac{\cos C}{11}$

Q. 5. Using Principle of Mathematical Induction prove that,

for all $n \geq 1$, $7 + 77 + 777 + 7777 + \text{- - -} + n$ *terms* $= \dfrac{7}{9}\left\{\dfrac{10(10^n - 1) - 9n}{9}\right\}$

Q. 6. In a survey of **100** students, the number of students studying the various languages were found to be **:** English only **18,** English but not Hindi **23,** English and Sanskrit **8,** English **26,** Sanskrit **48,** Sanskrit and Hindi **8,** no language **24.** Find **(*i*)** How many students were studying Hindi**?**
(*ii*) How many students were studying English and Hindi**?**

4 – Marks

Q. 7. Find the domain and range of the functions $f(x) = \sqrt{4x - x^2}$

Q. 8. Let **R** be a relation from **Q** to **Q** defined by $R = \{(a, b) : a - b \in Z$ and $a, b \in Q\}$. Are the following true?
(*i*) $(a, a) \in R$, for all $a \in Q$ **(*ii*)** $(a, b) \in R \Rightarrow (b, a) \in R$ **(*iii*)** $(a, b) \in R, (b, c) \in R \Rightarrow (a, c) \in R$.

Q. 9. For three sets ***A, B*** and ***C*** prove that
$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

Q. 10. A survey shows that **63%** of Indians like coffee, whereas **76%** likes tea. If ***x* %** of Indians like both coffee and tea, find the range of possible values of ***x*.**

Q. 11. Find the value of $\tan\left(\dfrac{\pi}{8}\right)$

Q. 12. Solve for x **:** $\tan x + \tan\left\{x + \dfrac{\pi}{3}\right\} + \tan\left\{x + \dfrac{2\pi}{3}\right\} = 3$

Q. 13. Using Principle of Mathematical Induction prove that**,** for all $n \geq 1$, $(2n + 7) < (n + 3)^2$.

Q. 14. Using Principle of Mathematical Induction prove that**,**

for all $n \geq 1$, $1.3 + 3.5 + 5.7 + \text{- - -} + (2n - 1)(2n + 1) = \dfrac{n(4n^2 + 6n - 1)}{3}$

Q. 15. If $x - iy = \sqrt{\dfrac{a - ib}{c - id}}$, then prove that $(x^2 + y^2)^2 = \dfrac{a^2 + b^2}{c^2 + d^2}$

Q. 16. Find real **θ** such that $\frac{3 + 2i\sin\theta}{1 - 2i\sin\theta}$ is purely imaginary.

Q. 17. Solve the inequation graphically: $2x + 3y \geq 6$, $x - 2y \leq 2$, $3x + 2y < 12$, $2y - 3x \leq 3$, $x \geq 0$, $y \geq 0$

Q. 18. How many litres of water will have to be added to **1125** litres of the **45%** solution of acid so that the resulting mixture will contain more than **25%** but less than **30%** acid content **?**

Q. 19. Solve the inequation : $\frac{2x + 4}{x - 1} \geq 5$; $x \in R$

Q. 20. What is the number of ways of choosing **3** cards from a pack of **52** playing cards**?** In how many of these **(*i*)** three cards are of the same suit**,** **(*ii*)** three cards belong to three different suits**.**

Q. 21. Find the number of words with or without meaning which can be made using all the letters of the word ***AGAIN*****.** If these words are written as in a dictionary**,** what will be the 50^{th} word**?**

Q. 22. Show that the middle term in the expansion of $(1 + x)^{2n}$ is $\frac{1.3.5.7.\ldots.(2n-1)2^n.\ x^n}{n!}$; $n \in Z_+$.

*****