BACKUP – 2A | **Mathematics – XII** | Max Marks – 80

Time – 3 hr

3 – Marks

$Q.1.$ Evaluate: $\int \left\{ log\,(log\,x) + \frac{1}{(log\,x)^2} \right\} dx$

$Q.2.$ Evaluate: $\int \frac{(3\,sin\,x - 2)\,cos\,x}{5 - cos^2 x - 4\,sin\,x}\,dx$

$Q.3.$ Find the area bounded by the curves, $y = x^2 + 2, y = x, x = 0\ \&\ x = 3$.

$Q.4.$ Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.

$Q.5.$ Show that the general solution of the differential equation $\frac{dy}{dx} + \frac{y^2 + y + 1}{x^2 + x + 1} = 0$ is given by $(x + y + 1) = A(1 - x - y - 2xy)$

$Q.6.$ Solve the differential equation: $x\frac{dy}{dx} + y - x + xy\,cot\,x = 0$

$Q.7.$ If a unit vector $\vec{a}$ makes angles $\frac{\pi}{3}$ with i, $\frac{\pi}{4}$ with j and an acute angle θ with k, then find θ and hence the vector $\vec{a}$.

$Q.8.$ Let $\vec{a} = i + 4j + 2k, \vec{b} = 3i - 2j + 7k, \vec{c} = 2i - j + 4k$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, & $\vec{c}.\vec{d} = 15$.

$Q.9.$ Let $\vec{\alpha} = 3i - j, \vec{\beta} = 2i + j - 3k$, then express $\vec{\beta}$ in the form of $\vec{\beta} = \vec{\beta_1} + \vec{\beta_2}$ where $\vec{\beta_1}$ is parallel to $\vec{\alpha}$ and $\vec{\beta_2}$ is perpendicular to $\vec{\alpha}$.

$Q.10.$ Find the distance between the lines : $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}; \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$

$Q.11.$ Consider the experiment of tossing a coin. If the coin shows head, toss it again but if it shows tail, then throw a die. Find the conditional probability of the event that 'the die shows a number greater than 4' given that 'there is at least one tail'.

$Q.12.$ A coin is biased so that the head is three times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails.

$Q.13.$ Solve the linear programming problem graphically:

$Maximise\ Z = 5x + 3y$, subjected to $3x + 5y \le 15, 5x + 2y \le 10, x \ge 0, y \ge 0$

$Q.14.$ Solve the linear programming problem graphically:

$Minimise\ \&\ Maximise\ Z = 5x + 10y$, subjected to $x + 2y \le 120, x + y \ge 60, x \ge 2y, x \ge 0, y \ge 0$.

$Q.15.$ Solve the linear programming problem graphically:

$Maximise\ Z = -x + 2y$, subjected to the constraints: $x \ge 3, x + y \ge 5, x + 2y \ge 6, y \ge 0$.

5 – Marks

$Q.16.$ Evaluate : $\int_0^{\pi} log(1 + cos\,x)dx = -\pi log2$

$Q.17.$ Evaluate : $\int_{-1}^{3/2} |x sin(\pi x)|dx$

$Q.18.$ Using integration, find the area of region $\{(x, y): 0 \le y \le x^2 + 1, 0 \le y \le x + 1, 0 \le x \le 2\}$

$Q.19.$ Solve the differential equation: $(x^3 - 3xy^2)dx = (y^3 - 3x^2y)dy$

$Q.20.$ A line makes angles $\alpha, \beta, \gamma\ and\ \delta$ with the four diagonals of a cube, prove that

$$cos^2\alpha + cos^2\beta + cos^2\gamma + cos^2\delta = \frac{4}{3}$$

$Q.21.$ Show that the lines $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}; \frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$ are coplanar, also find the equation of plane containing both the lines.

$Q.22.$ A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both diamonds. Find the probability of the lost card being a diamond.

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