BACKUP – 1B | **Mathematics – XII** | **Max Marks – 80**

Time – 3 hr

5 – Marks

(1) Solve the system of linear equations using matrix method,

$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4,\ \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1,\ \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$

(2) The cost of 4 kg onion, 3 kg wheat & 2 kg rice is Rs 60. The cost of 2 kg onion, 4 kg wheat & 6 kg rice is Rs 90. The cost of 6 kg onion, 2 kg wheat & 3 kg rice is Rs 70. Find the cost of each item per kg by matrix method.

(3) Find the greatest volume of the cylinder which can be inscribed in a right circular cone of height h and semi-vertical angle α.

(4) Show that semi-vertical angle of right circular cone of given surface area and maximum volume is $sin^{-1}\left(\frac{1}{3}\right)$

(5) If length of three sides of a trapezium other than base are equal to 10cm, then find the area of the trapezium when it is maximum.

(6) Show that the function $f: R \longrightarrow (-1\ 1)$ defined by $f(x)=\frac{x}{1+|x|}$ is bijective.

(7) Find the point on the curve $9y^2 = x^3$, where the normal to the curve makes equal intercepts with the axes.

3 – Marks

(8) Check whether the relation R defined over real numbers as $R=\{(a,b): a \leq b^3\}$ is reflexive, symmetric or transitive

(9) Show that $f: N \to N$, given by $f(x)=\begin{cases} x+1\ ;\ x\ is\ odd \\ x-1\ ;\ x\ is\ even \end{cases}$ is both one-one and onto.

(10) If R_1 and R_2 are two equivalence relations, then show that $R_1 \cap R_2$ is also an equivalence relation.

(11) Show that : $sin^{-1}\left(\frac{3}{5}\right)+cos^{-1}\left(\frac{12}{13}\right)=sin^{-1}\left(\frac{56}{65}\right)$

(12) Simplify : $sin^{-1}\left(2x\sqrt{1-x^2}\right)\ ;\ \frac{1}{\sqrt{2}}<x<1$

(13) Show that: $sin^{-1}\left(\frac{12}{13}\right)+cos^{-1}\left(\frac{4}{5}\right)+tan^{-1}\left(\frac{63}{16}\right)=\pi$

(14) Find x if, $tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}tan^{-1}(x), x>0$

(15) Show that : $cot^{-1}\left\{\frac{\sqrt{1+sin\,x}+\sqrt{1-sin\,x}}{\sqrt{1+sin\,x}-\sqrt{1-sin\,x}}\right\}=\frac{x}{2}\ ,\ x\in\left[0\ \frac{\pi}{2}\right]$

(16) If, $A=\begin{bmatrix} 0 & -tan\left(\frac{\alpha}{2}\right) \\ tan\left(\frac{\alpha}{2}\right) & 0 \end{bmatrix}$ and $I=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ Show that, $(I+A)=(I-A)\begin{bmatrix} cos\,\alpha & -sin\,\alpha \\ sin\,\alpha & cos\,\alpha \end{bmatrix}$

(17) Find the matrix X so that $X.\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\2 & 4 & 6\end{bmatrix}$

(18) Find the values of k so that the function $f(x)=\begin{cases}\frac{k\cos x}{\pi-2x}\ ;\ x\neq\frac{\pi}{2}\\ 3\ \ \ \ \ \ ;\ x=\frac{\pi}{2}\end{cases}$ is continuous at $x=\frac{\pi}{2}$.

(19) If $y^x+x^y+x^x=a^b$ then find $\frac{dy}{dx}$

(20) If $y=e^{a\,cos^{-1}x}\ ;\ -1<x<1$, show that, $(1-x^2)y_2-x\,y_1-a^2\,y=0$.

(21) If $y=sin^{-1}x+sin^{-1}\sqrt{1-x^2}\ , -1<x<1$. Find $\frac{dy}{dx}$.

(22) If $y=\frac{cos^3 t}{\sqrt{\cos 2t}}, x=\frac{sin^3 t}{\sqrt{\cos 2t}}$, prove that $\frac{dy}{dx}=-cot\,3t$

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