TEST PAPER – 3
Vivekanand School Special
Mathematics – XI

Time : 3 hr **Max Marks : 90**

General Instructions :

1. All questions are compulsory.
2. The question paper consists of **28 questions** divided into three sections **A, B** and **C.** **Section A** comprises of **10** questions of **one mark** each, **Section B** comprises of **14** questions of **four marks** each and **Section C** comprises of **04** questions of **six marks** each.
3. All questions in **Section A** are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in **04** questions of **four marks** each and **01** questions of **six marks** each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION – A

Q. 1. Find the number of all possible relations that can be defined over the set **{1, 2}**

Q. 2. Find the range of the function $f(x) = \frac{x^2}{x^2 + 1}$

Q. 3. Find the coordinates of the foot of perpendicular from the point **(–1, 3)** to the line $3x - 4y - 16 = 0$.

Q. 4. The perpendicular from the origin to the line $y = mx + c$ meets it at the point **(–1, 2).** Find the values of m and c.

Q. 5. Find the coefficient of x^9 in the expansion $(9 - 6x^3 + x^6)^3$

Q. 6. Find r, if $5P(4, r) = 6P(5, r - 1)$

Q. 7. Three vertices of a parallelogram **ABCD** are **A(3, – 1, 2), B (1, 2, – 4)** and **C(– 1, 1, 2).** Find the coordinates of the fourth vertex.

Q. 8. Find the ratio in which the line segment joining the points **(4, 8, 10)** and **(6, 10, – 8)** is divided by the YZ – plane.

Q. 9. If $y = \frac{\sqrt{x}}{\cos x}$, Find $\frac{dy}{dx}$

Q. 10. Evaluate : $\lim_{x \to 0} \left\{ \frac{x\left(e^{2+x} - e^2\right)}{1 - \cos x} \right\}$

P.T.O

SECTION – B

Q. 11. Let **R** be a relation from **N** to **N** defined by $\mathbf{R} = \{(a, b) : a, b \in \mathbf{N} \text{ and } a = b^2\}$. Are the following true?
(*i*) $(a, a) \in \mathbf{R}$, for all $a \in \mathbf{N}$ **(*ii*)** $(a, b) \in \mathbf{R} \Rightarrow (b, a) \in \mathbf{R}$ **(*iii*)** $(a, b) \in \mathbf{R}, (b, c) \in \mathbf{R} \Rightarrow (a, c) \in \mathbf{R}$.

Q. 12. Prove that : $\dfrac{\sin x}{\cos 3x} + \dfrac{\sin 3x}{\cos 9x} + \dfrac{\sin 9x}{\cos 27x} = \dfrac{\tan 27x - \tan x}{2}$

Q. 13. Solve for A : $\tan A + \tan\left\{A + \dfrac{\pi}{3}\right\} + \tan\left\{A + \dfrac{2\pi}{3}\right\} = 3.\tan 3A$

OR

Show that , $\cot 3x \cdot \cot 5x - \cot 5x \cdot \cot 8x - \cot 8x \cdot \cot 3x = 1$

Q. 14. Determine the **5** card combination out of a deck of **52** cards if at least one of the **5** cards has to be king**?**

Q. 15. What will be the rank of the word **INDIAN,** if letters of the word are written as in a dictionary**?**

OR

How many natural number not exceeding **4321** can be formed with the digits **1, 2, 3,** and **4,** if the digits can repeat**?**

Q. 16. Two students Anil and Ashima appeared in an examination**.** The probability that Anil will qualify the examination is **0.05** and that Ashima will qualify the examination is **0.10.** The probability that both will qualify the examination is **0.02.** Find the probability that**,**
(*i*) Both Anil and Ashima will not qualify the examination**.**
(*ii*) At least one of them will not qualify the examination.
(*iii*) Only one of them will qualify the examination**.**

Q. 17. On her vacations Veena visits four cities **(A, B, C** and **D)** in a random order**.** What is the probability that she visits **(*i*) A** before **B ?** **(*ii*) A** just before **B ?** **(*iii*) A** before **B** and **B** before **C?**

Q. 18. Find the equation of hyperbola having foci **(± 4 , 0)** and the length of latus rectum is **12.**

Q. 19. Find the coordinates of foci and vertices**,** eccentricity length of latus rectum of the ellipse $\dfrac{x^2}{4} + \dfrac{y^2}{36} = 1$

OR

The cable of uniform loaded suspension bridge hangs in the form of a parabola**.** The roadway is horizontal and **100** *m* long is supported by vertical wire attached to the cable**,** the longest wire being **30** *m* and the shortest wire being **6** *m***.** Find the length of the wire attached to the roadway **18** *m* from the middle**.**

Q. 20. Find the coordinate of the point(s) which trisect the segment joining the points **(2, 1, –3)** and **(5, –8, 3)**

Q. 21. Show that the equation of the lines passing through the origin and making an angle θ with the line $y = mx + c$ are $\dfrac{y}{x} = \dfrac{m + \tan\theta}{1 - m.\tan\theta}$, and $\dfrac{y}{x} = \dfrac{m - \tan\theta}{1 + m.\tan\theta}$

Q. 22. Using *ab – initio* find the derivative of the function $f(x) = e^{\sqrt{\sin x}}$

Q. 23. If, $f(x) = \begin{cases} |x| + 1 & , x < 0 \\ 0 & , x = 0 \\ |x| - 1 & , x > 0 \end{cases}$ For what value(s) of a does $\lim\limits_{x \to a} f(x)$ exists?

Q. 24. Show that the coefficient of the middle term in the expansion of $(1 + x)^{2n}$ is equal to the sum of the coefficients of two middle terms in the expansion of $(1 + x)^{2n-1}$.

OR

If a and b are distinct integers, using binomial theorem prove that $a - b$ is a factor of $a^n - b^n$, $\forall\ n \in Z_+$

SECTION – C

Q. 25. Find the distance of the point **(1 , 2)** from the line $4x + 7y + 5 = 0$ measured along the line $2x - y = 3$

Q. 26. (*i*) A solution of **8%** boric acid is to be diluted by adding a **2%** boric acid solution to it . The resulting mixture is to be more than **4%** but less than **6%** boric acid. If we have **640** litres of the **8%** solution, how many litres of the **2%** solution will have to be added?

(*ii*) A man wants to cut three lengths from a single piece of board of length **91** *cm* .The second length is to be **3** *cm* longer than the shortest and the third length is to be twice as long as the shortest. What are the possible lengths of the shortest board if the third piece is to be at least **5** *cm* longer than the second ?

Q. 27. In certain locality of a town of **10,000** families, it was found that **40%** families buy newspaper **A, 20 %** families buy newspaper **B** and **10%** families buy newspaper **C. 5%** families buy **A** and **B, 3%** families buy **B** and **C** and **4%** families buy **A** and **C.** If **2%** families buy all the three newspaper, find the number of families which buy**. (*i*)** exactly one news paper
(*ii*) exactly two news paper
(*iii*) No news paper**.**

Q. 28. From the data given below state which group is more variable**, A** or **B?**

Marks	**10 – 20**	**20 – 30**	**30 – 40**	**40 – 50**	**50 – 60**	**60 – 70**	**70 – 80**
Group A	**9**	**17**	**32**	**33**	**40**	**10**	**9**
Group B	**10**	**20**	**30**	**25**	**43**	**15**	**7**

OR

Calculate Standard Deviation, using short-cut method for the following distribution**:**

Classes	**0 – 30**	**30 – 60**	**60 – 90**	**90 – 120**	**120 –150**	**150 –180**	**180 -210**
Frequency	**2**	**3**	**5**	**10**	**3**	**5**	**2**

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