TEST PAPER – 3

Green Fields Pb. School Special

Mathematics – XI

Time : 3 hr **Max Marks : 100**

General Instructions :

1. All questions are compulsory.
2. The question paper consists of **29 questions** divided into three sections **A, B** and **C.** **Section A** comprises of **10** questions of **one mark** each, **Section B** comprises of **12** questions of **four marks** each and **Section C** comprises of **07** questions of **six marks** each.
3. All questions in **Section A** are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in **04** questions of **four marks** each and **02** questions of **six marks** each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION – A

Q. 1. If $\cos(\alpha + \beta) = \frac{4}{5}$ and $\sin(\alpha - \beta) = \frac{5}{13}$, where α, β lie between **0** and $\frac{\pi}{4}$, then find $\tan 2\alpha$.

Q. 2. Solve for natural x : $\left|\frac{x-3}{6}\right| \geq \frac{5}{2}$

Q. 3. Find the equation of ellipse having $b = 3, c = 4,$ centre at the origin and foci on a *x – axis*.

Q. 4. Find the points on the *x – axis*, whose distances from the line $\frac{x}{3} + \frac{y}{4} = 1$ are 4 units.

Q. 5. Write the contrapositive of the statement ***p : x is an even number implies that x is divisible by* 4.**

Q. 6. If $z_1 = 2 - i$, $z_2 = -2 + i$, find Re $\left(\frac{z_1 z_2}{\overline{z_1}}\right)$

Q. 7. Find r, if $P(n, r) = C(n, r)$

Q. 8. A letter is chosen at random from the word **'*ASSASSINATION*'.** Find the probability that letter is a vowel.

Q. 9. Find the middle term in $\left\{\frac{3}{x^2} - \frac{x^3}{6}\right\}^8$

Q. 10. Find the equation of the set of points which are equidistant from the points **(1, 2, 3)** and **(3, 2, –1).**

P.T.O

SECTION – B

Q. 11. In a survey of **400** students in a school**, 100** were listed as taking apple juice**, 150** as taking orange juice and **75** were listed as taking both apple as well as orange juice. Find how many students were taking neither apple juice nor orange juice.

OR

If **P(A)** is power set of set **A,** then prove that $\mathbf{P(A \cap B) = P(A) \cap P(B)}$**.**

Q. 12. If the relation R in the set A of points in a plane given by
R = {(P, Q) : distance of the point **P** from the origin = the distance of the point **Q** from the origin**},**
(*i*) $(P, P) \in R$, for all $P \in A$ (*ii*) $(P, Q) \in R \Rightarrow (Q, P) \in R$
(*iii*) $(P, Q) \in R, (Q, S) \in R \Rightarrow (P, S) \in R$.

Q. 13. If $\sin A + \sin B = a$; $\cos A + \cos B = b$, then prove that, $\tan\left(\frac{A-B}{2}\right) = \frac{\sqrt{4 - a^2 - b^2}}{\sqrt{a^2 + b^2}}$

OR

Prove that : $\sin 20^\circ \cdot \sin 40^\circ \cdot \sin 60^\circ \cdot \sin 80^\circ = \frac{3}{16}$

Q. 14. If *a, b, c* are in **A.P** prove that $\frac{1}{\sqrt{b}+\sqrt{c}}$, $\frac{1}{\sqrt{c}+\sqrt{a}}$, $\frac{1}{\sqrt{a}+\sqrt{b}}$ are also in **A.P**

OR

If $a\left(\frac{1}{b} + \frac{1}{c}\right), b\left(\frac{1}{c} + \frac{1}{a}\right), c\left(\frac{1}{a} + \frac{1}{b}\right)$ are in **A.P** prove that a, b, c are also in **A.P**

Q. 15. Convert the complex number $\frac{1+3i}{1-2i}$ into polar form.

Q. 16. If for the function $f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x} & , x \neq \frac{\pi}{2} \\ 3 & , x = \frac{\pi}{2} \end{cases}$, we have $\lim_{x \to \pi/2} f(x) = f\left(\frac{\pi}{2}\right)$. Find the values of k

Q. 17. Two students Anil and Ashima appeared in an examination**.** The probability that Anil will qualify the examination is **0.05** and that Ashima will qualify the examination is **0.10.** The probability that both will qualify the examination is **0.02.** Find the probability that**,**
(*i*) Both Anil and Ashima will not qualify the examination**.**
(*ii*) At least one of them will not qualify the examination.

Q. 18. Solve the following inequalities graphically**:** $5x + 10y \leq 50$, $x + y \geq 3$, $x - y < 0$, $y \leq 4$, $x \geq 0$, $y \geq 0$

Q. 19. Find the coordinates of the point which divides the line segment joining the points **(1, –2, 3)** and **(3, 4, –5)** in the ratio **2 : 3** (*i*) internally (*ii*) externally**.**

Q. 20. In how many ways can a cricket team be selected from a group of **25** players containing **10** batsmen**, 8** bowlers**, 5** all rounders and **2** wicket keepers**.** Assume that the team of **11** players requires **5** batsmen**, 3** all rounders**, 2** bowlers and **1** wicket keeper**.**
If a player is involved in match fixing of cricket match what values does he losses.

Q. 21. Check whether the statement ***p : If x, y∈ Z are such that x and y are odd, then xy is odd***, is true or not.

Q. 22. If in the expansion of $(1 + x)^n$, the coefficient of 5^{th} , 6^{th} and 7^{th} terms are in **A.P.** Find ***n*****.**

OR

Find the value of **'*k*'** so that the term independent of *x* in $\left\{\sqrt{x} + \frac{k}{x^2}\right\}^{10}$ is **405.**

SECTION – C

Q. 23. If $2\tan A = 3\tan B$, prove that, $\tan(A - B) = \frac{\sin 2B}{5 - \cos 2B}$

OR

In a triangle **ABC** prove that : $(b - c)\cot\left(\frac{A}{2}\right) + (c - a)\cot\left(\frac{B}{2}\right) + (a - b)\cot\left(\frac{C}{2}\right) = 0$

Q. 24. Using first principle, find the derivative of the function $f(x) = \log(\sin x)$

Q. 25. Using principle of mathematical induction prove that,

for all $n \geq 1$, $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \text{- - -} + n \text{ terms} = \frac{n(n+1)^2(n+2)}{12}$

Q. 26. Find the distance of the point **(1, 2)** from the line $4x + 7y + 5 = 0$ along the line $2x - y = 3$

Q. 27. Find the centre, the lengths of axes, eccentricity, foci of the hyperbola: $x^2 - 2y^2 - 2x + 8y - 1 = 0.$

OR

A beam is supported at its ends by supports which are **12** *m* apart. Since the load is concentrated at its centre, there is a deflection of **3** *cm* at the centre and the deflected beam is in the shape of a parabola. How far from the centre is the deflection **1** *cm*?

Q. 28. Find the sum of the series up to *n terms* : $1^3 + \frac{1^3 + 2^3}{1 + 3} + \frac{1^3 + 2^3 + 3^3}{1 + 3 + 5} + \text{- - -}$

Q. 29. The mean and standard deviation of **100** observations were calculated as **40** and **5.1,** respectively by a student who took by mistake **50** instead of **40** for one observation. What are the correct mean and standard deviation?

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