

TEST PAPER – 1 (Green Ways Modern)

Mathematics – XI

Time : 3 hr

Max Marks : 100

GENERAL INSTRUCTIONS :-

1. All questions are compulsory.
2. SECTION – A comprises of 6 questions of one marks each.
3. SECTION – B comprises of 13 questions of four marks each.
4. SECTION – C comprises of 7 questions of six marks each.
5. Internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

SECTION – A

- Q. 1. Find the number of all possible relations that can be defined from the set $\{1, 2, 3\}$ on the set $\{4, 5\}$.
- Q. 2. Find the 4th term from end in the expansion $(9 - 12x + 4x^2)^{12}$
- Q. 3. If the foot of perpendicular of $(1, 2)$ in a line is $(3, 6)$, find the equation of the line.
- Q. 4. Find the negation of the statement “The sum of 3 and 4 is 9”.
- Q. 5. Find the components of the statement “24 is a multiple of 2, 4 and 8”.
- Q. 6. For the statement “To enter a country, you need a passport or a voter registration card” determine whether an inclusive “Or” or exclusive “Or” is used. Give reasons for your answer.

SECTION – B

- Q. 7. A college awarded 38 medals in football, 15 in basketball and 20 in cricket. If these medals went to a total of 58 men and only three men got medals in all the three sports, how many received medals in exactly two of the three sports ?

- Q. 8. By using the principle of mathematical induction prove that,

$$\text{for all } n \geq 1, \quad 1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \dots + \frac{1}{1+2+3+\dots+n} = \frac{2n}{n+1}$$

- Q. 9. Let R be a relation from N to N defined by $R = \{(a, b) : a, b \in \mathbb{N} \text{ and } a = b^2\}$. Are the following true?
(i) $(a, a) \in R$, for all $a \in \mathbb{N}$ (ii) $(a, b) \in R, \Rightarrow (b, a) \in R$ (iii) $(a, b) \in R, (b, c) \in R \Rightarrow (a, c) \in R$.

- Q. 10. Solve the system of inequalities graphically: $x + 2y \leq 10, x + y \geq 1, x - y \leq 0, x \geq 0, y \geq 0$

- Q. 11. Convert the complex number, $\frac{i-1}{\cos(\frac{\pi}{3}) + i \sin(\frac{\pi}{3})}$, into polar form

- Q. 12. Prove that the middle term of the expansion $(1 + 4x + 4x^2)^n$ is $\frac{1.3.5.7. \dots (2n-1)4^n \cdot x^n}{n!}$; $n \in \mathbb{Z}_+$

- Q. 13. If p, q, r are in G.P. and the equations, $px^2 + 2qx + r = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then show that $\frac{d}{p}, \frac{e}{q}, \frac{f}{r}$ are in A.P

OR

If $p^{\text{th}}, q^{\text{th}}, r^{\text{th}}$ and s^{th} terms of an A.P are in G.P, then show that $(p - q), (q - r), (r - s)$ are also in G.P.

- Q. 14. Find the equation of hyperbola having foci $(\pm 4, 0)$ and the length of latus rectum is 12.

OR

Find the equation of the hyperbola having foci on $(0, \pm \sqrt{10})$ and which passes through $(2, 3)$.

- Q. 15. Find the coordinates of the point where the line through $(3, -4, -5)$ and $(2, -3, 1)$ crosses the plane $2x + y + z = 7$.

- Q. 16. Find the number of words with or without meaning which can be made using all the letters of the word ZOOLOGY. If these words are written as in a dictionary, what will be the 50th word?

- Q. 17. Let A and B be sets. If $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$ for some set X, show that $A = B$.

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Q. 18. Find the direction in which a line must be drawn through the point $(-1, 2)$ so that its point of intersection with the line $x + y = 4$ may be at a distance 3 units from this point .

OR

Find the image of the point $(3, 8)$ with respect to the line $x + 3y = 7$ assuming the line to be a plane mirror.

Q. 19. Prove that, $\sin^3 x + \sin^3 \left\{x + \frac{2\pi}{3}\right\} + \sin^3 \left\{x + \frac{4\pi}{3}\right\} = -\frac{3}{4} \sin 3x$

OR

If in a triangle ABC, $\frac{b+c}{12} = \frac{c+a}{13} = \frac{a+b}{15}$, prove that $\frac{\cos A}{2} = \frac{\cos B}{7} = \frac{\cos C}{11}$

SECTION - C

Q. 20. In a triangle ABC prove that : $(b-c) \cot\left(\frac{A}{2}\right) + (c-a) \cot\left(\frac{B}{2}\right) + (a-b) \cot\left(\frac{C}{2}\right) = 0$

OR

In a triangle ABC prove that : $a^3 \sin(B-C) + b^3 \sin(C-A) + c^3 \sin(A-B) = 0$

Q. 21. Find the value of $\sin\left(\frac{x}{2}\right)$, $\cos\left(\frac{x}{2}\right)$ and $\tan\left(\frac{x}{2}\right)$ if $\sin x = \frac{1}{4}$; x lies in IIrd quadrant .

Q. 22. In how many ways can the letters of the word *PERMUTATIONS* be arranged if the
(i) No two vowels are not together (ii) there are always 5 letters between P and S?

Q. 23. If 4-digit numbers greater than 5,000 are randomly formed from the digits 0, 1, 3, 5, and 7, what is the probability of forming a number divisible by 5 when,
(i) the digits are repeated ? (ii) the repetition of digits is not allowed ?

Q. 24. Show that: $\frac{1 \times 2^2 + 2 \times 3^2 + \dots + n \times (n+1)^2}{1^2 \times 2 + 2^2 \times 3 + \dots + n^2 \times (n+1)} = \frac{3n+5}{3n+1}$

Q. 25. The mean and standard deviation of 20 observations are found to be 10 and 2, respectively. On rechecking, it was found that an observation 8 was incorrect. Calculate the correct mean and standard deviation in each of the following cases:

(i) If wrong item is omitted. (ii) If it is replaced by 12.

Q. 26. Find the differential coefficient of $\sin^3 \sqrt{x}$, by first principle.

OR

If $\lim_{x \rightarrow \pi/2} f(x) = f\left(\frac{\pi}{2}\right)$, for the function is $f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x}, & x \neq \frac{\pi}{2} \\ k, & x = \frac{\pi}{2} \end{cases}$ find the value of k .
